1. Find disjunctive and conjunctive normal forms, construct the relevant Karnaugh maps and hence compute optimal circuits for the following.

   (a) \( a_1 = (x_1 \land x_2) \lor (((\overline{x_3} \lor x_1) \land \overline{x_2}) \lor x_2) \)

   (b) \( a_2 = (x_1 \lor ((x_1 \land x_2) \lor (\overline{x_1} \land \overline{x_2} \land \overline{x_3}))) \land x_3 \)

   (c) \( a_3 = x_1 \), considered as a boolean expression in the variables \( \{x_1, x_2, x_3\} \).

2. Describe a boolean function corresponding to the logic network below.

   ![Logic Network Diagram]

3. Negate and simplify:

   (a) \( p \rightarrow \forall y(q \land Bx) \)

   (b) \( \forall x \exists y \exists d \forall t \exists q(r \lor \exists m(n \rightarrow \overline{q})) \)

4. Prove that *nand* is functionally complete. That is if we let \( p \ast q \) mean \( \neg(p \land q) \) show that the other connectives, \( \land \), \( \lor \), \( \neg \) and \( \rightarrow \) are expressible in terms of *.

**Tutorial Exercises for the Week 28 July–1 August**

1. Compute miniterm, maxiterm and optimal expressions corresponding to the expressions with the given Karnaugh maps.

   (a) 
   | x  | yz | y\overline{z} | \overline{y}z | \overline{y}z |
   |----|----|----------------|----------------|
   | \( x \) | +  | +              | +              | +              |
   | \( \overline{x} \) | +  |                | +              |               |
2. A nor gate is

\[ p \quad \text{nor} \quad q \]

Sketch a logical network equivalent to the one below consisting of only nor gates.

\[ \begin{array}{cccc}
q & q\bar{r} & \bar{q}\bar{r} & \bar{q}r \\
p & + & + & + \\
\bar{p} & & + & \\
\end{array} \]

\[ \begin{array}{cccc}
p\bar{q} & p\bar{q} & \bar{p}q & pq \\
x\bar{y} & + & + & + \\
\bar{x}\bar{y} & + & + & + \\
x\bar{y} & + & + & + \\
\end{array} \]