Assignment 1

Due: 22 July

1. Construct truth-tables for the following propositions, and determine if each is a tautology, a contradiction, or is contingent:
   (i) \( p \land \neg (q \lor p) \)
   (ii) \( (p \to q) \to \neg (p \lor \neg q) \)
   (iii) \( (p \to (q \land r)) \to (\neg r \to \neg p) \)

2. Negate the following propositions and then simplify the result:
   (i) \( (\neg (p \land q)) \to r \)
   (ii) \( \forall x \exists y (p \land \neg r) \)
   (iii) \( \forall x \in A ((x > 0) \to \exists y \in B (x \leq y)) \)

   **Note:** “Simplify” means repeatedly replace expressions by logically equivalent ones until all negation symbols are moved as far as possible to the right.

3. Use properties of logical equivalence to show that
   \[ ((p \to q) \to q) \equiv (p \lor q) \]

4. Translate into symbols using the indicated dictionary. The universe is the collection of people. \( xLy \) means “\( x \) loves \( y \)” \( xPy \) means “\( x \) is the parent of \( y \)”
   (i) Everyone loves themselves.
   (ii) Everyone loves someone.
   (iii) Someone loves everyone.
   (iv) Everyone loves their parents.
   (v) People love their grandchildren.
   (vi) Nobody loves someone who loves themselves.
Tutorial Exercises for the Week 19-22 July

1. Construct truth-tables for the following propositions, and determine if each is a tautology, a contradiction, or is contingent:

\[ q \leftrightarrow (\neg q \land p), \quad \neg q \rightarrow ((q \rightarrow p), \quad (p \rightarrow q) \lor r, \quad p \rightarrow (q \rightarrow r). \]

2. Negate and simplify

\[ p \rightarrow (q \land r), \quad \forall x(p \lor \neg q), \quad \forall x \in B(p \rightarrow \exists y \in C(q \lor r)). \]

3. Use properties of logical equivalence to show that

\[ ((p \rightarrow q) \land p) \rightarrow q \]

is a tautology.