Assignment 2 due 26 July

1. Find disjunctive and conjunctive normal forms.
   (i)  \(a_1 = (x_1 \land x_2) \lor ((x_1 \lor x_2) \land x_2) \lor x_2\)
   (ii) \(a_2 = (x_1 \lor ((x_1 \land x_2) \lor (x_1 \lor x_2 \land x_3)) \land x_3\)
   (iii) \(a_3 = (x_1 \lor x_2) \land x_1 \land \overline{x_2}\)

   Marks 2, 2, 1

2. Prove that nor is functionally complete. That is if we let \(p \ast q \) mean \(\neg(p \lor q)\) show that the other connectives, \(\land, \lor, \neg\) and \(\to\) are expressible in terms of \(\ast\).

   Marks 3

3. Negate and simplify
   (i)  \(\forall x(p \rightarrow \forall y (q \land By)) \land r\)
   (ii) \(\forall x \exists y \exists z \forall t (r \lor \forall m (p \leq q))\)

   Marks 4

4. Using the properties of logical equivalence, prove that \((P \land (P \to (Q \to R))) \to (Q \to R)\) is a tautology.

   Marks 2

5. Let \(Pxy\) mean that “\(x\) prefers \(y\)”. Let \(Ox\) mean that “\(x\) is old” and \(Nx\) mean that “\(x\) is new”, and \(Qx\) mean that “\(x\) is a person” and \(Hx\) mean that “\(x\) is a house”. Let \(a\) be Adam and \(b\) be Betty. Using only this dictionary translate the following into predicate logic.

   (i) Betty is old.
   (ii) All old people prefer new houses.
   (iii) Some young (i.e. not old) people prefer new houses.
   (iv) Every old person prefers some old house.

   Marks 2

6. Prove by a direct argument that if \(x\) and \(y\) are odd integers, then \(x + y\) is even.

   Marks 2

7. Let \(x\) be an integer. Prove by contrapositive that if \(x^2\) is odd, then \(x\) is odd.

   Marks 2
Tutorial Exercises for the Week 21 July-26 July

1. A nand gate is one for $\overline{x y}$.
   Prove that nand is functionally complete. That is if we let $p \ast q$ mean $\neg(p \land q)$ show that the
   other connectives, $\land$, $\lor$, $\neg$ and $\rightarrow$ are expressible in terms of $\ast$.

2. Negate and simplify
   (a) $\forall x \exists y \exists z (r \rightarrow (s \rightarrow \overline{p} \lor q))$
   (b) $(r \lor (s \rightarrow (q \rightarrow (v \rightarrow \forall m (a \lor b))))$)

3. Find disjunctive and conjunctive normal forms.
   (i) $a_1 = (x_1 \lor x_2) \lor ((\overline{x_3} \rightarrow x_1) \land \overline{x_2}) \lor x_2$
   (ii) $a_2 = (x_1 \land ((x_1 \land x_2) \rightarrow (\overline{x_1} \land \overline{x_2} \land \overline{x_3}))) \land x_3$

4. Let $x$ and $y$ be integers. Prove by contrapositive that if $xy$ is odd, then $x$ and $y$ are odd.

5. Is the following argument valid?
   If Wellington is in New Zealand, then Paris is not in France. But Paris is in France.
   Therefore Wellington is not in New Zealand.

6. Do some of the miscellaneous translations into predicate logic on the 161 home page.