As you see on the assignment other connectives

\[(\phi)\]

\[\text{biconditional}\]

\[\text{iff}\]

\[A \iff B\]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>(A \iff B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
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</tbody>
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\[(A \iff B) \equiv (A \to B) \land (B \to A)\]

Lots of other connectives
but we won't study

\( \text{nand} \) and \( \text{nor} \)

\[
A \ (\text{nand}) \ B \equiv \neg(A \land B)
\]

\[
A \ (\text{nor}) \ B \equiv \neg(A \lor B)
\]

\[
\neg A \equiv A \ (\text{nand}) \ A
\]

---

returning to

properties of \( \equiv \)

\[
\begin{array}{c|c|c|c|c|c}
S & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
\[ \Diamond \Rightarrow (\Box \vee \Diamond) \]

\[ \equiv \neg \Box \vee (\neg \Box \vee \neg (\Box \vee \Box)) \]

\[ \equiv \neg \Box \vee (\Box \vee \Box) \}

\[ \equiv (\Box \vee \Box) \}

\[ \equiv \neg \Box \vee (\Box \vee \Box) \}

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\[ \equiv (\Box \vee \Box) \}

\[ \equiv (\Box \vee \Box) \} \]
Sometimes there is a quick method to determine validity.

"Assigning values"

\[ q \lor (\neg \phi \rightarrow \neg \psi) \]

"How could this be false?"

Both sides must be false.

\[ q = 1 \]

Also \( p \rightarrow (p \lor q) \) to be false.

\[ p = 1 \land p \lor q = 0 \]

(\[ if \this \ is \ not \ helpful \ forget \it\])
Quantifiers

\[ \forall x \quad \text{"for all } x \quad \text{"} \]

\[ \exists x \quad \text{"there is an } x \quad \text{such that} \quad \text{"} \]

Examples:

\[ P(x) = x \text{ is prime} \]

\[ P_x \]

\[ N_x = \text{"} x \text{ is a natural num} \]
There are two numbers whose sum is prime.

\[ \exists x \exists y \left( N_x \land N_y \land \left( p_x \land p_y \land p_{x+y} \right) \right) \]

Negate quantifiers

\[ \neg \forall x \equiv \exists x \neg ( \neg ) \]

\[ \equiv \forall x \neg ( \neg ) \]
People

\( \text{universe} = \text{in the room} \)

\( H_x = x \text{ has hair} \)

\( Q_x = x \text{ is over 50} \)

\( A \forall y, z \) has the same age as.

\((\exists x) (H_x)\)

\[\underline{\text{did not specify universe.}}\]

\((\exists x \in \text{R})(H_x)\)  \(\text{R = room, people} \)
Everyone in the room over so has hair

\[(\forall x \in R) \ (Q_x \rightarrow H_x)\]

\[\neg \neg (Q_x \land H_x)\]

\[\neg \neg (Q_x \land H_x) = (\forall x \in R) (Q_x \land H_x)\]
3. Classical Quantification Theory

(i) Some laws are good
   \( Lx = x \) is a law, \( Gy = y \) is good

(j) Some people admire anyone who is rich and influential
   \( Px = x \) is a person, \( Ry = y \) is rich, \( Lz = z \) is influential, \( yAz = \text{admiries } z \)

(k) If some laws are unjust then someone will suffer
   \( Lz = z \) is a law, \( Uy = y \) is unjust, \( Px = x \) is a person, \( Sz = z \) will suffer

(l) Everyone who lives in Brisbane lives in Australia
   \( b = \text{Brisbane}, a = \text{Australia}, Px = x \) is a person, \( yLx = y \) lives in \( x \)

(m) Every building in Melbourne is in Australia
   \( m = \text{Melbourne}, a = \text{Australia}, Bx = z \) is a building, \( yIx = \text{in } x \)

(n) Every person lives at some address
   \( Px = x \) is a person, \( Ay = y \) is an address, \( yDz = y \) lives at \( z \)

(o) No students are protesters
   \( Sy = y \) is a student, \( Px = z \) is a person

(p) Someone respects someone
   \( Px = x \) is a person, \( yRx = y \) respects \( z \)

(q) Someone respects everyone
   \( as \) for \( (p) \)

(r) Someone respects no one
   \( as \) for \( (p) \)

(s) All students are protesters
   \( as \) for \( (o) \)

(t) Everyone loves somebody sometime
   \( Px = x \) is a person, \( Tz = x \) is a time, \( Lyz = y \) loves \( z \) at \( x \)

2. Supplying a suitable dictionary, translate the following into QT:

(a) Nothing is worthwhile

(b) Something is rotten

(c) Something is rotten in the state of Denmark

(d) Only old things are valuable

(e) Every cloud has a silver lining

(f) No clouds are welcome

(g) No clouds bring rain unless they are saturated

(h) Crime never pays

(i) All whales are extinct

(j) No whales exist

(k) He steals not his shadow who faces the sun

(l) Only the brave deserve the fair

(m) Apples and oranges are delicious and nutritious

(n) Everyone respects someone or other

(o) There is some one person whom everyone respects

(p) A platypus has fur, beak, claws, and lays eggs

(q) No one is perfect

(r) If anyone can do logic, Jack can

(s) If all material objects are in space and time then everything not in space or time is not a material object

(t) All cold, grey, stone buildings are depressing

Given the following dictionary only, translate the English sentences into QT:

\[ Mx \quad \text{x is a man} \]
\[ Hx \quad \text{x is a human} \]
\[ xSy \quad \text{x is married to y} \]
\[ xPy \quad \text{x is a parent of y} \]
\[ a \quad \text{Arthur} \]
\[ b \quad \text{Bertha} \]
\[ r \quad \text{Robin} \]

(a) Arthur is married to Bertha

(b) Someone is married to Bertha

(c) Whoever is married to Bertha is a man

(d) All men are human

(e) Arthur and Bertha are Robin's parents

(f) Arthur is Robin's father

(g) Robin is Arthur's offspring

(h) Robin is Arthur's son

(i) Robin is Bertha's daughter

(j) Arthur is a grandparent of Robin

(k) If two people are married, one is male and one is female

(l) Arthur and Robin have a common parent

(m) Arthur is a parent

(n) Every parent is married

(o) Robin is an unmarried mother

(p) If Arthur is Robin's father-in-law then Bertha is his mother-in-law

(q) Of any three people, if the second is the father-in-law of the first and is married to the third, then the third is the mother-in-law of the first

(r) Arthur and Bertha are brother and sister

(s) Arthur is Bertha's father or uncle

(t) Bertha is Robin's maternal grandmother
\( a \Pr \land b \Pr \)

\((\exists x) \,(a \Pr \land x \Pr)\)

\((\forall x) \,(P_x) \quad (\exists y) \,(P_y)\)

\((\forall x) \,(P_x \rightarrow (\exists y) \,(P_y \Delta))\)

\((\forall x) \,(P_x \rightarrow \exists y \exists t \,(P_y \Delta t))\)
"Negate & simplify"

\( \neg (\forall x \exists y \in R \rightarrow \exists z \forall t (\forall s)) \)

\[\exists (\exists x) \neg (\exists y \in R \rightarrow \exists z \forall t (\forall s)) \]

\[\exists (\exists x) (\forall y \in R) \neg (\forall t \exists z (\forall s))\]

\[\exists (\exists x) (\forall y \in R) (\forall t \forall z (\forall s))\]

\[\exists (\exists x) (\forall y \in R) (\forall t \forall z (\forall s)) \]

\[\exists (\exists x) (\forall y \in R) (\forall t \forall z (\forall s)) \]