Miscellaneous solutions attached to
the end of
today notes
for translations.
Mathematical Induction

Dominoes

Claim: They all fall over.

To do this it is enough to show

1. Domino 1 falls.
2. Sufficiently close that if domino \( k \) falls knock over domino \( k + 1 \)
Principle of Mathematical Induction

1. To prove \( P(n) \) holds for all \( n \in \{1, 2, 3, \ldots\} \), it is enough to prove:
   1. \( P(1) \) holds
   2. For all \( k \):
      - If \( P(k) \) holds, then \( P(k+1) \) holds.

This is the Induction Hypothesis.
\[ RTP \]

\[ P(n) : 0 + 1 + 2 + \ldots + n = \frac{n(n+1)}{2} \]

\[ \forall n \geq 0 \]

\[ P(1) : 0 = 1 \]

\[ P(k) : 0 + 1 + 2 + \ldots + k = \frac{k(k+1)}{2} \]

\[ P(k+1) = 0 + 1 + 2 + \ldots + k + (k+1) = \frac{(k+1)(k+2)}{2} \]
Basis

\[ n = 0 \]

\[ P(0) : \quad 0 = \frac{0(0+1)}{2} = \frac{0}{2} = 0 \]

Induction step

\[ \forall k \left( P(k) \Rightarrow P(k+1) \right) \]

RTP if \( \frac{1}{2} \)

\[ \text{If } 0 + 1 + 2 + \ldots + k = \frac{k(k+1)}{2} \]

\[ \therefore 0 + 1 + 2 + \ldots + k + 1 = \frac{(k+1)(k+2)}{2} \]

Assume (*)

\[ 0 + 1 + 2 + \ldots + (k+1) \]

\[ = 0 + 1 + 2 + \ldots + k + (k+1) \]

\[ = \frac{k(k+1)}{2} + k + 1 = \frac{k(k+1) + 2(k+1)}{2} \]

\[ = \frac{(k+1)(k+2)}{2} \]
Hypothesis was

0 + 1 + - + k = \frac{k(k+1)}{2}

Assume Hyp

derive conclusion
Claim

only have 3c & 5c stamps.

Claim possible to make up any postage of 8c or more.

e.g. 9c = 3 \times 3c

10c = 2 \times 5c.

P(n) any postage \geq 8c can be done with 3c & 5c stamps.
Basis

\[ n = 8 \]

\[ 8^c = 3^c + 5^c \quad \text{starp} \quad \checkmark \]

Induction step:

RTP if I can do \( k^c \)

I can do \( (k+1)^c \)

\( P(k) \): I can do \( k^c \)

Assume \( P(k) \)

Show how to do \( (k+1)^c \)

Case 1: I used some \( 5^c \) stamp to make \( k^c \)

Take \( 5^c \) stamp out.

Replace with 2 \( 3^c \) stamps.
Can't no 5¢ stamps were used to get k¢.

i.e. only used 3¢ stamps.

as \( k \geq 8 \), need at least 3 + 3¢ stamps.

Take 3×3¢ out replace with 2×5¢.
Tromino

We say an $n \times n$ board is deficient if it misses one square.
Claim

$2^m$ for all $m \geq 1$

any $2^m \times 2^m$ deficient board can be tiled by trominoes

$p(m)$
By $n=1$ basis
any $2 \times 2$ board is a domino.

Induction step.
Assume $m=k$.

Consider $p(k+1)$.
A $2^{k+1} \times 2^{k+1}$ board.

$P(k)$ say this can be tiled with dominoes.

Remove these. Makes the others deficient. Can apply $P(k)$. 

deticient
(i) Some laws are good
   \((Lz = z)\) is a law, \((Gy = y)\) is good

(j) Some people admire anyone who is rich and influential
   \((Px = x)\) is a person, \((Ry = y)\) is rich, \((Lz = z)\) is influential, \((yAz\) admires \(z)\)

(k) If some laws are unjust then someone will suffer
   \((Lz = z)\) is a law, \((Uy = y)\) unjust, \((Px = x)\) is a person, \((Sz = z)\) suffer

(l) Everyone who lives in Brisbane lives in Australia
   \((b = \text{Brisbane}, a = \text{Australia}, Px = x)\) is a person, \((yLx = y\) lives in \(x)\)

(m) Every building in Melbourne is in Australia
   \((m = \text{Melbourne}, a = \text{Australia}, Bz = z\) is a building\((, yLx = y\) lives in \(x)\)

(n) Every person lives at some address
   \((Px = x)\) is a person, \((Ay = y)\) is an address, \((yDz = y\) lives at \(z)\)

(o) No students are protesters
   \((Sx = x)\) is a student, \((Pz = z)\) is a protester

(p) Someone respects someone
   \((Px = x)\) is a person, \((yRx = y\) respects \(z)\)

(q) Someone respects everyone
   \((as\) for \((p))\)

(r) Someone respects no one
   \((as\) for \((p))\)

(s) All students are protesters
   \((as\) for \((o))\)

(t) Everyone loves somebody sometime
   \((Pz = z)\) is a person, \((Tx = x)\) is a time, \((Lyz = y\) loves \(z\) at \(x)\)

2. Supplying a suitable dictionary, translate the following into QT.
   (a) Nothing is worthwhile
   (b) Something is rotten
   (c) Something is rotten in the state of Denmark
   (d) Only old things are valuable
   (e) Every cloud has a silver lining
   (f) No clouds are welcome
   (g) No clouds bring rain unless they are saturated
   (h) Crime never pays
   (i) All whales are extinct
   (j) No whales exist
   (k) Helen's not his shadow who faces the sun
   (l) Only the brave deserve the fair
   (m) Apples and oranges are delicious and nutritious
   (n) Everyone respects someone or other

(o) There is some one person whom everyone respects
(p) A platypus has fur, beak, claws, and lays eggs
(q) No one is perfect
(r) If anyone can do logic, Jack can
(s) If all material objects are in space and time then everything not in space or time is not a material object
(t) All cold, grey, stone buildings are depressing

Given the following dictionary only, translate the English sentences into QT:

\[
\begin{align*}
Mx & \rightarrow x \text{ is a man} \\
Hx & \rightarrow x \text{ is a human} \\
xSy & \rightarrow x \text{ is married to } y \\
xPy & \rightarrow x \text{ is a parent of } y \\
a & \rightarrow \text{ Arthur} \\
b & \rightarrow \text{ Bertha} \\
r & \rightarrow \text{ Robin} \\
\end{align*}
\]

(a) Arthur is married to Bertha
(b) Someone is married to Bertha
(c) Whoever is married to Bertha is a man
(d) All men are human
(e) Arthur and Bertha are Robin's parents
(f) Arthur is Robin's father
(g) Robin is Arthur's offspring
(h) Robin is Arthur's son
(i) Robin is Bertha's daughter
(j) Arthur is a grandparent of Robin
(k) If two people are married, one is male and one is female
(l) Arthur and Robin have a common parent
(m) Arthur is a parent
(n) Every parent is married
(o) Robin is an unmarried mother
(p) If Arthur is Robin's father-in-law then Bertha is his mother-in-law
(q) Of any three people, if the second is the father-in-law of the first and is married to the third, then the third is the mother-in-law of the first
(r) Arthur and Bertha are brother and sister
(s) Arthur is Bertha's father or uncle
(t) Bertha is Robin's maternal grandmother

\[
\begin{align*}
(\forall x)(\forall c)(xSc \Rightarrow Mx) \\
(\forall x)(\forall c)(Nc \Rightarrow \neg Mx)
\end{align*}
\]
(1) \( (\exists x) (Lx \land Gx) \)
(2) \( (\exists x) (Pz \land (\forall y) (Py \land Rx) \rightarrow \exists y) \)
(3) \( (\exists y) (Ly \land Rx) \rightarrow (\exists x) (Rx \land Sz) \)
(4) \( (\forall x) (Sx \land Rx) \rightarrow (\exists y) (Ly \land Rx) \)
(5) \( (\forall x) (Bx \land xIa) \rightarrow (\exists y) (Py \land xLy) \)
(6) \( (\forall x) (Bx \land xIa) \rightarrow (\exists y) (Py \land xLy) \)
(7) \( (\forall x) (Sx \rightarrow Rz) \)
(8) \( (\exists x) (Bx \land (\forall y) (Py \land x Ry)) \)
(9) \( (\exists x) (Bx \land (\forall y) (Py \land x Ry)) \)
(10) \( (\exists x) (Bx \land (\forall y) (Py \land x Ry)) \)
(11) \( (\forall x) (Sz \rightarrow Pz) \)
2. (a) \( \exists x \quad (x = x \land x = x) \) is a thing.
   \( \forall x \quad \text{worthwhile} \)
   \( (\forall x) \quad (T_x \rightarrow \neg W x) \).

3. \( R x : x \text{ is rook} \)
   \( (\exists x) \quad (T x \land R x) \)

(c) \( d = \text{demeh} \).
   \( x \land \neg \neg j \)
   \( \exists x \quad (T x \land R x \land x \land d) \)

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(a) \( a \not\leq b \)
(b) \( (\exists x) \quad (x \not\leq b) \)
(c) \( (\exists x) \quad (x \not\leq b \rightarrow M x) \).
(d) \( (\forall x) \quad (M x \rightarrow H x) \)
(e) \( a \land b \land b \land a \land c \land b \land c \)
(f) \( a \land c \land M a \)
(g) \( a \land c \)
(h) \( \exists x \land \neg \neg j \)

(r) \( (\exists y) \quad (a \land y) \)
(s) \( (\exists x) \quad (y \land x) \)
(t) \( (\forall x) \quad (\neg x \land \neg x) \)
(u) \( (\exists z) \quad (\neg c \land \neg c) \)
(v) \( \neg \neg j \land \neg \neg j \)
(w) \( \neg \neg j \land \neg \neg j \)
(x) \( \neg \neg j \land \neg \neg j \)