1. (i) What properties should an equivalence relation possess?

reflexive, symmetric, transitive.

(ii) Let $R$ be a relation on \{1, 2, 3, 4, 5\} given by

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (4, 5), (5, 4), (2, 5), (5, 2)\}.$$ 

Explain why $R$ is not an equivalence relation.

Because it's not transitive.
2. Let $A = \{1, 2, 3, \ldots, 7\}$ and $B = \{a, e, i, o, u\}$. Let $R$ be a relation on $A$ given by

$$R = \{(x, y) \in A \times A \mid x \text{ is a factor of } y\} \tag{1.2} \text{ and } S,$$

and $S$ be a relation from $A$ to $B$ defined by

$$S = \{(2, a), (3, e), (5, u), (7, u)\}.$$

(i) Draw the Hasse diagram of $R$.

(ii) Complete

(a) $S^{-1} = \{(a, 2), (e, 3), (u, 5), (u, 7)\}.$

$$RS = \{(1, a), (1, e), (1, u), (2, a), (3, e), (5, u), (7, u)\}.$$
3. (i) Suppose \( f : \mathbb{R} \to \mathbb{R} \) is given by \( f(x) = \frac{1}{x} \). Explain why \( f \) is not a function.

Because when \( x = 0 \), \( \frac{1}{0} \) is undefined.

(ii) Suppose \( f : \{a, b, c, d\} \to \{a, b, c, d\} \) is given by

\[
f(a) = a, f(b) = b, f(c) = c, f(d) = c.
\]

(a) Explain why \( f \) is not one-to-one.

because both \( f(c) \) and \( f(d) \) equal to \( c \).

(b) Explain why \( f \) is not onto.

Because \( d \) is not included.

(iii) (a) Find the number of all maps from \( A = \{1, 2, 3\} \) to \( B = \{a, b, c, d\} \).

\[
4 \times 4 \times 4 = 4^3 \text{ functions}
\]

(b) Find the number of injective maps from \( A = \{1, 2, 3\} \) to \( B = \{a, b, c, d\} \).

As before, but decreasing options

\[
4 \times 3 \times 2 = 4!
\]
4. (i) Find the gcd of 68 and 20.

\[
\begin{align*}
68 &= 20 \times 3 + 8 \\
20 &= 8 \times 2 + 4 \\
8 &= 4 \times 2 \\
\end{align*}
\]

\[
\therefore \quad \text{gcd}(68, 20) = 4.
\]

(ii) Find integers \( n, m \) so that \( 17m + 5n = 1 \).

\[
\begin{align*}
17 &= 5 \times 3 + 2 \\
5 &= 2 \times 2 + 1 \\
\end{align*}
\]

\[
\text{gcd}(17, 5) = 1
\]

\[
\therefore \quad 1 = 5 - 2 \times 2.
\]

\[
\begin{align*}
1 &= 5 - (17 - 5 \times 3) \times 2 \\
&= 5 \times 7 - 17 \times 2 \\
&= 5 \times 7 + (17 \times (-2))
\end{align*}
\]

\[
\therefore \quad 17 \times (-2) + 5 \times 7 = 1
\]

\[
m = -2, \quad n = 7.
\]
(iii) Find the lcm (least common multiple) of 64 and 24.

\[ 64 = 2 \times 32 = 2^2 \times 16 = 2^2 \times 2^4 = 2^6 \]

\[ 24 = 2 \times 12 = 2^2 \times 6 = 2^3 \times 3 \]

\[ \therefore \text{lcm} = 2^6 \times 3^1. \]