Those Properties
from last time

De Morgan, commutativity
Dist. can be used
to "prove" things

Can derive A from B
(expressions) by a
series of equivalences
& substitution
$\neg \text{ to prove}$

$$P \rightarrow (Q \rightarrow P)$$

$$\neg P \rightarrow (A \rightarrow B) \equiv (\neg A \lor B)$$

$$\neg P \rightarrow (\neg A \lor B)$$

$$\equiv \neg P \lor (\neg A \lor B)$$

$$\equiv \neg P \lor (P \lor \neg A)$$

$$\equiv (\neg P \lor P) \lor \neg A$$

$$\equiv \text{True} \lor \neg A$$

$\equiv \text{True}$
Same method:

\[ A \land (P \rightarrow (P \lor Q)) \]

\[ = A \land (P \lor (P \land Q)) \]

\[ = A \land ((P \lor P) \land (P \land Q)) \]

\[ = A \land (P \land Q) \]

\[ = T \lor (P \land Q) \] comm

\[ = (A \lor \neg P) \lor Q \] assoc.

\[ = T \lor \neg P \equiv T \]

\[ A \land (B \lor C) \equiv (A \land B) \lor (A \land C) \]

\[ A \lor (B \land C) \equiv (A \lor B) \land (A \lor C) \]
Propositional logic is rather limited in expressability.

Many many logics tailored towards stronger expressibility

First order or predicate logic

Notation

\[ P \times P(x) \]

is a property of an individual \( x \)
\( x \) is a propositional variable \( P_x \) is
is a predicate or
propositional function

\( P_x : \ " x \text{ is prime} \"

\( P_x : x > 0 \) unary

\( P_{xy} : x > y \)

Binary predicate

\( L_{xy} : " x \text{ loves } y \" \)
$\forall x$ only assumes a truth value when it is "instantiated" ie put in a value for $x$ 

"$x$ is prime"

we enrich logic by adding quantifiers

$(\forall x) (P_x) = "\text{for all } x, \ P_x \text{ holds}"$

$(\exists x) (P_x) = "\text{there is an } x \text{ such that } P_x \text{ holds}"$
If the universe changed
all people need an additional relation
\[ R_x = \sim x \text{ is in the room} \]

\[ (\exists x) (R_x \land \neg H_x) \]

"nobody in the room over so has hair"

\[ (\forall x) (O_x \rightarrow \neg H_x) \]

\[ \neg (\exists x) (O_x \land H_x) \]
Example

Universe = people in the room

\( H_x = "x \text{ has hair} \) \\
\( D_x = "x \text{ is over 50} \) \\
\( A_{xy} = "x \text{ is same age as } y \) \\
\( T_{xyz} = "x \text{ is a tall as either } y \text{ or } z \) \\

\((\exists x) \ (H_x) = \text{ somebody in the room has hair}\)
Everyone who has the same age are the same height.

∀x (∀y (Axy))
∀x ∀y (Axy → (Txyy ∨ Tjxx))

Order matters

∀x ∃y. (x Ay) True
∃x) ∃y. (x Ay) False
Care about negation

\neg (\forall x) \neg \neg (\neg x)

\equiv (\exists x) \neg (\neg x)

\equiv (\exists x) \neg (\neg x)

\equiv (\forall x) \neg (\neg x)

\equiv "\text{negate & simplify}"
\[(\forall x)(\exists y)(x \rightarrow (\exists z)(z \rightarrow (Rxy \rightarrow g)))\]

\[\exists (\forall x)(\forall y) \rightarrow (\exists z)(\exists t)(Rxy \rightarrow g)\]

\[\exists (\forall x)(\forall y)(R \land \neg (\forall z)(\exists t)(Rxy \rightarrow g))\]

\[\neg (A \rightarrow B) \equiv (A \land \neg B)\]

\[\exists (\forall x)(\forall y)(R \land (\exists z)(\exists t)(Rxy \rightarrow g))\]

\[\exists (\forall x)(\forall y)(R \land (\exists z)(\exists t)(Rxy \land g))\]

\[\exists (\forall x)(\forall y)(R \land (\exists z)(\exists t)(Rxy \land 0))\]

\[\exists (\forall x)(\forall y)(R \land (\exists z)(\exists t)(Rxy \land 0 \land 0))\]
Notation

\((\forall x > 2)\) (___)
in stead of

\((\forall x)\) (\(x > 2 \rightarrow \) (___))

instead of

\((\forall x \in \mathbb{R})\) (___)

instead of

\((\forall x)\) (\(x \in \mathbb{R} \rightarrow \) (___))

Laziness