1. Propositional logic
   - Truth tables
   - Tautology, contingent contradiction

2. Predicate logic
   - CNF & DNF
   - Translations
     "Negate & Simplify"

* Truth trees
Proof strategies

- Direct proofs
- Proofs by contrapositive

To prove \( p \implies q \)

Show \( \neg q \implies \neg p \).

- Proofs by contradiction of tree trees
  Assume \( \neg A \)
  Get a contradiction
  \( \therefore A \)
Induction

\[ \rightarrow 1 \]

"Set Theory" \\
and counting \\
up to I/O

\[ 1. \text{ Truth Trees } \]

\[ \text{Predicate Logic} \]

\[ \text{Induction with } \leq \]
\[(\forall x) \ (f_x \Rightarrow g_{\forall x}) \]
\[\Rightarrow \ (\forall y) \ (f_y \Rightarrow (\forall x) \ (y)) \]

\[\exists y \ H_y \]

\[H_b \ni \text{"realizes" } H_y. \]

\[\exists y \ (P_z \ & \ Q_z) \]

\[P_b \ & \ Q_b \]

\[\exists x \ (M_x) \]

\[M_a \]

\[P_a \ & \ Q_a \]
1. \( (\forall x)(F_x \rightarrow G_x) \rightarrow (\forall x)(F_x \rightarrow \forall x)G_x \)

\[ \neg (A \rightarrow B) \]

2. \( A (\forall x)(F_x \rightarrow G_x) \)  \text{ from 1}

3. \( \neg B \rightarrow (\forall x)(F_x \rightarrow (\forall x)G_x) \)

4. \( (\forall x)(F_x) \)  \text{ from 3}

5. \( \neg (\forall x)(G_x) \)

6. \( (\exists x) \neg G_x \)  \text{ from 5}

7. \( \neg G(a) \)  \text{ instantiation}

8. \( \neg F_a \)  \text{ from 4}

9. \( F_a \rightarrow G_a \)

10. \( G_a \)  \text{ \(8,9,\text{ MP.} \) and \(2 \)}
Ga.

$$ (Ax) (Ey) (Rx y) $$

$$ (Ey) (Ray) $$

$$ Rab $$

$$ Rbc $$

$$ (Ey) (Rby) $$

$$ Rdc $$

$$ (Ey) (Rdy) $$

$$ etc.$$
\[ (\forall q)(\neg Faq \lor \neg Fbq) \]

10. Dem.

11. \( \neg Faq \lor \neg Fba \) \( \rightarrow q \)

12. \( \neg Fab \lor \neg Fbb \) \( \rightarrow b \)

13. \( \neg Fac \lor \neg Fbc \) \( \neg c \)
\[ = 4 (\exists x)(\exists y)(\exists z) \neg ((F_{xz} \land 
abla y) \rightarrow (\exists y)(F_{xy} \land F_{yz})) \]

5. \[ \rightarrow (Fca \land Fcb) \rightarrow \]

6. \[ \exists y (Fca \land \neg Fcb) \] from 5.

7. \[ \exists y (Fca \land \neg Fcb) \] from 5.

8. \[ Fca \rightarrow Fcb \] from 6

9. \[ Fca \rightarrow Fcb \] from 6

10. \[ (\forall y) \neg (Fca \land \neg Fcb) \] from 7.
1. \((\forall x)(\forall y)(\forall z) ((F_{xz} \land F_{zy}) \implies F_{xy})\)

2. \((\forall x)(\forall y)(\forall z) ((F_{x\bar{z}} \land F_{yz}) \implies (F_{xy} \lor F_{yx}))\)

3. \((\forall x)(\exists y) F_{xy}\)

\[\implies \forall y \exists z \exists x \big( (F_{x\bar{z}} \land F_{yz}) \implies (F_{xy} \lor F_{yx}) \big) \]

\[\forall \theta (F_{\theta x} \lor F_{\theta y}) \n(C)\]
$W_x = "x is worthwhile"

$\forall x (W_x) \land \forall x (W_x)$

$R_x = "x is rotten"

$\exists x (R_x) \land \exists x (R_x)$

$D_x = "x is in Denmark"

$R_x = "x is rotten"

$(\exists x) (R_x \land D_x)$
$R \times y \equiv x \text{ respects } y$ \\

$(\forall x) (\exists y) (R \times y)$ \\

$=$ \\

$W_x = x \text{ is a whole}$ \\

$(\exists x = x \text{ exists}$ \\

$\subseteq (\exists y) (W_x)$ \\

$(\forall y) (W_x \rightarrow \exists x)$
3 Classical Quantification Theory

(i) Some laws are good
   \( Lz = z \) is a law, \( Gy = y \) is good
(j) Some people admire anyone who is rich and influential
   \( Px = x \) is a person, \( Ry = y \) is rich, \( Iz = z \) is influential, \( yAz = y \) admires \( z \)
(k) If some laws are unjust then someone will suffer
   \( Lz = z \) is a law, \( Uy = y \) is unjust, \( Px = x \) is a person, \( Sz = z \) will suffer
(l) Everyone who lives in Brisbane lives in Australia
   \( b = \text{Brisbane}, a = \text{Australia}, Px = x \) is a person, \( yLx = y \) lives at \( x \)
(m) Every building in Melbourne is in Australia
   \( m = \text{Melbourne}, a = \text{Australia}, Bz = z \) is a building, \( yIx = y \) in \( x \)
(n) Every person lives at some address
   \( Px = x \) is a person, \( Ay = y \) is an address, \( yDz = y \) lives at \( z \)
(o) No students are protesters
   \( Sy = y \) is a student, \( Pz = z \) is a protester
(p) Someone respects someone
   \( Px = x \) is a person, \( yRz = y \) respects \( z \)
(q) Someone respects everyone
   (as for \( p \))
(r) Someone respects no one
   (as for \( p \))
(s) All students are protesters
   (as for \( o \))
(t) Everyone loves somebody sometime
   \( Px = x \) is a person, \( Tx = x \) is a time, \( Lyxz = y \) loves \( z \) at \( x \)

2. Supplying a suitable dictionary, translate the following into QT.

(a) Nothing is worthwhile
(b) Something is rotten
(c) Something is rotten in the state of Denmark
(d) Only old things are valuable
(e) Every cloud has a silver lining
(f) No clouds are welcome
(g) No clouds bring rain unless they are saturated
(h) Crime never pays
(i) All whales are extinct
(j) No whales exist
(k) He sees not his shadow who faces the sun
(l) Only the brave deserve the fair
(m) Apples and oranges are delicious and nutritious
(n) Everyone respects someone or other
\( Mx = x \text{ is material object} \)
\( Sx = x \text{ is in space} \)
\( Tx = x \text{ is in time} \)

\((\forall x) (Mx \implies (Sx \lor Tx)) \)

\( \implies (\forall x) (\neg (\exists y \exists z (y \lor z \lor \neg w))) \)

\( \implies \neg MW \)