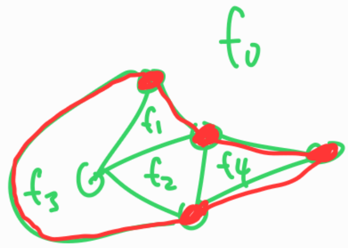
MATH361 | Lecture 14

Test on 15 April, 3:10pm

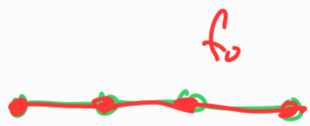
Recap: • <u>plane graph</u>: graph + planar embedding

- the embedding of the edges partitions the rest of the plane into arcwise-connected regions, called the <u>faces</u> of the plane graph

e.g.



- the <u>outer face</u> corresponds to the region not enclosed by a collection of edges.



- each face has a <u>boundary</u> that corresponds to a closed walk, but usually we think of the boundary as a subgraph.

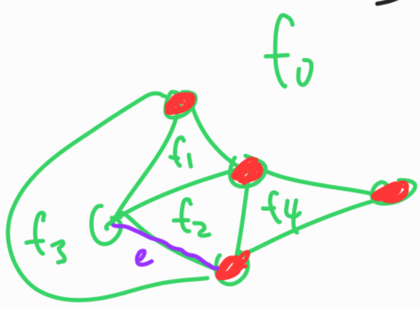
For a face f in a plane graph.

→ $\partial(f)$ refers the set of edges in the boundary of f

→ a vertex or edge is <u>incident</u> with f if it is in the boundary of f.

Two faces in a plane graph are <u>adjacent</u> if there is some edge that they are both incident to.

e.g.

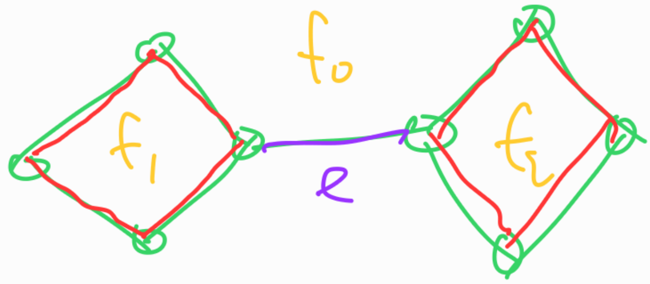


f_2 and f_3 are adjacent

e is incident to f_2 (and f_3).

Lemma 4.9 (iii) Let e be an edge in a plane graph G. If e is a bridge, then e is incident with only 1 face; Otherwise, e is incident with precisely 2 faces.

e.g.



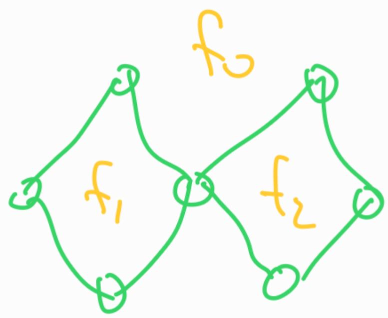
e is only incident with f_0.

The degree of a face f in a plane graph, denoted $d(f)$, is [# of edges in the boundary of f] + [# of bridges in the boundary of f]

e.g. $d(f_1) = 4$ and $d(f_0) = 10$.

The boundary of a face is often a cycle, but is not when the plane graph has a cut vertex.

e.g.

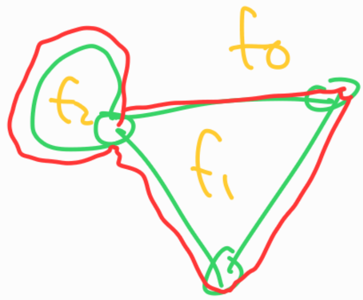


Theorem 4.13 (Whitney, 1932):

Let G be a loopless 2-connected plane graph. Then every face boundary is a cycle.

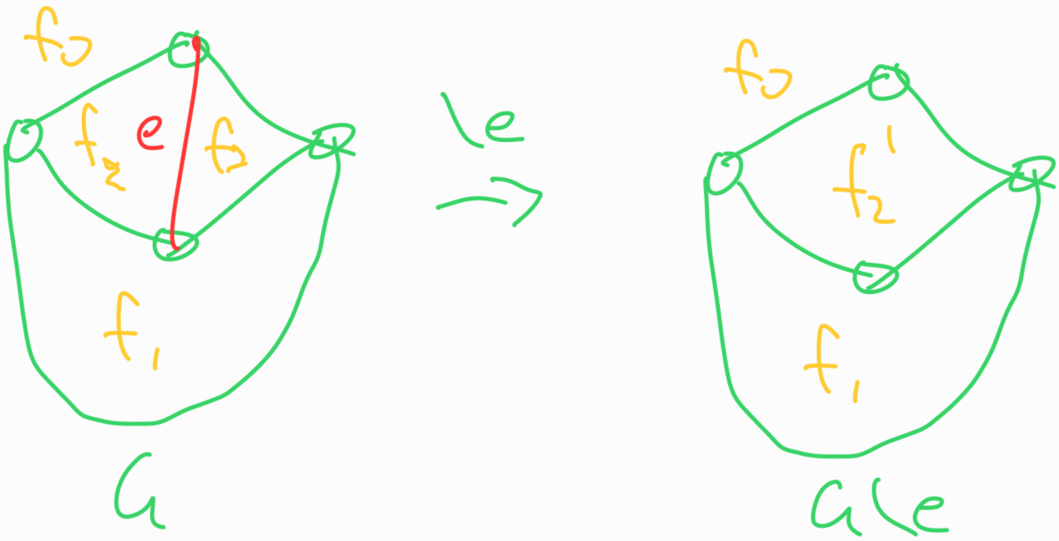
Needed

e.g.



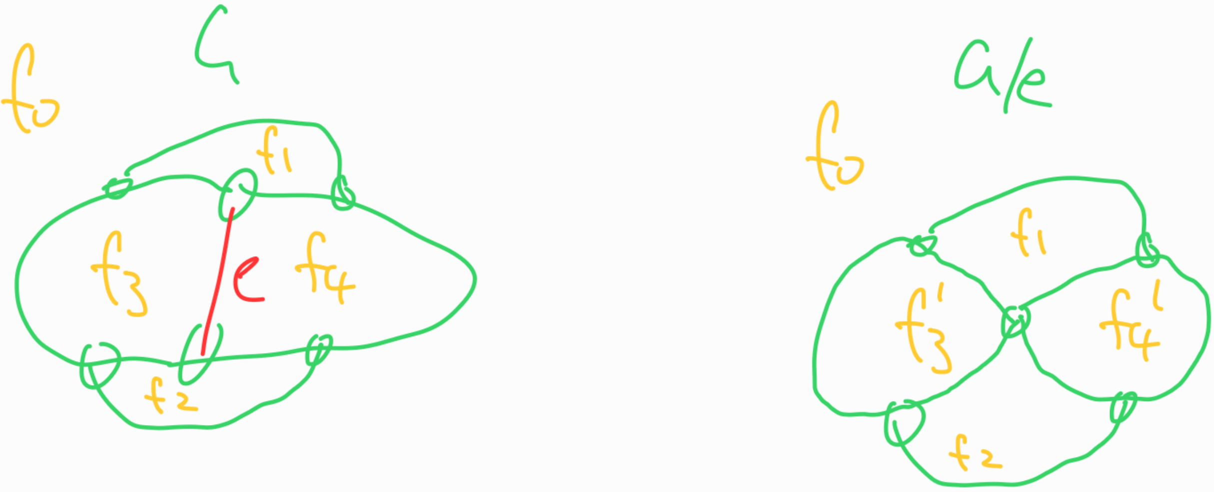
We can delete an edge from a plane graph and retain a planar embedding in the natural way

e.g.



What about contraction?

e.g.



<u>Lemma</u> 4.11: Let G be a plane graph, let $e \in E(G)$

where e is not a bridge, and let f_1 and f_2 be the faces incident with e. Then there is a planar embedding of G/e such that:

i) $\partial(f_1) \setminus \{e\}$ and $\partial(f_2) \setminus \{e\}$ are the edge sets of a face boundary

ii) for each $f \in F(G) \setminus \{f_1, f_2\}$,

$\partial(f)$ is the edge set of a face boundary.

<u>Corollary 4.12</u>: If G is a planar graph, then any minor of G is also planar.