MATH361 | Lecture 2

Recap: walk, path, Eulerian walk
loop, parallel edges, simple
incident, adjacent
degree

Handshaking lemma: for any graph $G=(V,E)$, $\sum_{v\in V} d(v) = 2|E|$.

Corollary 1.4: Every graph has an even number of vertices of odd degree.

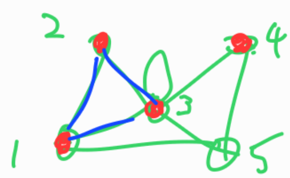
Proof: Suppose $G=(V,E)$ has an odd number of vertices of odd degree. Then $\sum_{v\in V} d(v)$ is odd. But $2|E|$ is even, so, by the Handshaking lemma, this is a contradiction. So every graph has an even number of vertices of odd degree. $\square$

Subgraphs Let $G=(V,E)$ be a graph.

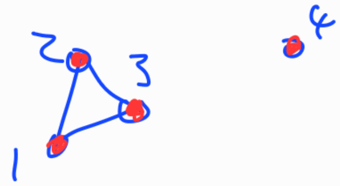
For a $F \subseteq E$, the vertices incident with F is the union of the vertices incident with each $e \in F$.

Let $V' \subseteq V$ and $E' \subseteq E$ such that the vertices incident with E' are contained in V'. Then the graph (V', E') is a subgraph of G.

e.g.



G



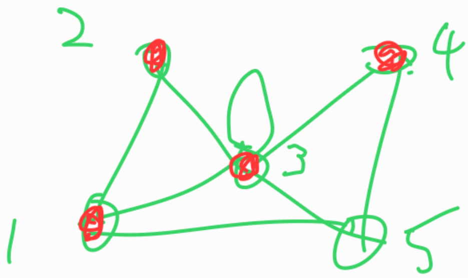
$H = (V', E')$

E' V'

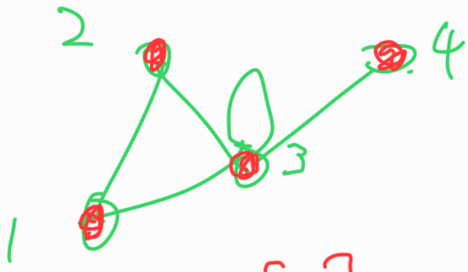
H is a subgraph of G.

Let $V' \subseteq V$. The induced subgraph $G[V']$ of G is the subgraph with vertex set V' and edge set consisting of edges of G with both ends contained in V'.

e.g.



G

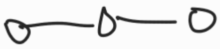


$G[X]$

$X = \{1, 2, 3, 4\}$

Note that H is not an induced subgraph of G.

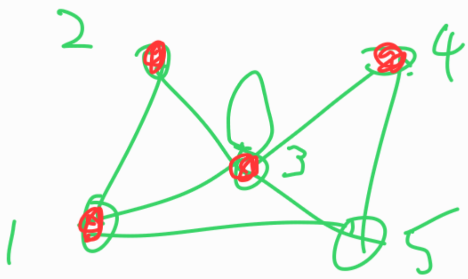
The graph P_n is the path graph on n vertices

 P_3

 P_4

etc.

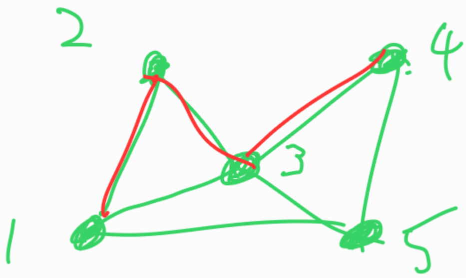
e.g.



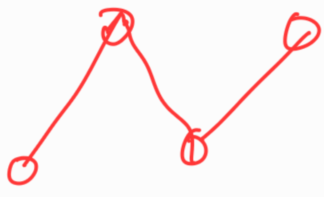
G contains P_4 as a induced subgraph, but not P_5.

Let $E' \subseteq E$. The edge-induced subgraph $G[E']$ of G is the subgraph of G with edge set E' and vertex set consisting of the vertices incident with E'.

eg.



G



$G[F]$

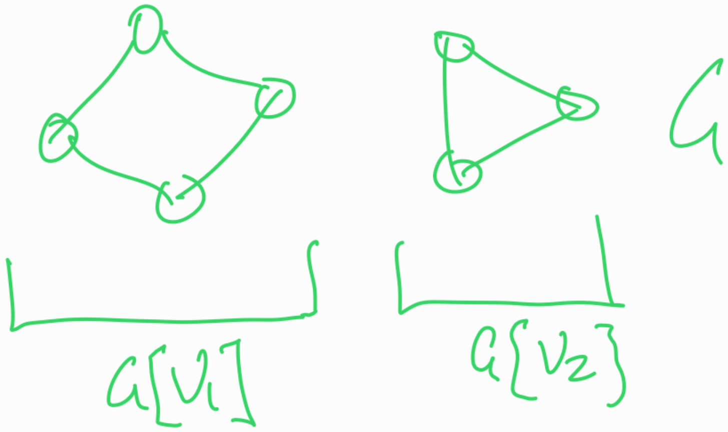
$F = \{12, 23, 34\}$

For a graph G with vertex set V, we define a relation $\sim$ where for $u, v \in V$, we have $u \sim v$ if and only if there is a walk from u to v.

Thm 1.5 $\sim$ is an equivalence relation.

Let $V_1, V_2, \ldots, V_t$ be the equivalence classes of V under $\sim$. Then $G[V_1], G[V_2], \ldots, G[V_t]$ are the components of G

e.g.



A graph is connected if it has precisely 1 component.

Equivalently: a graph is connected iff it has at least 1 vertex and there is a walk between any pair of vertices.

Lemma 1.1: If a graph has a walk W from a vertex u to a vertex v, then it has a path from u to v on a subset of the edges of W.

Proof left for assignment 1.

By Lemma 1.1: a graph is connected iff it has at least 1 vertex and there is a path between any pair of vertices.

A graph is disconnected if it has at least 2 components.

Cycles

Notation: for a graph we refer to the vertex set as $V(G)$ and the edge set as $E(G)$.

Let G be a graph. A cycle is a subgraph H of G with $V(H) = \{v_1, v_2, \dots, v_t\} \subseteq V(G)$ and $E(H) = \{e_1, \dots, e_t\} \subseteq E(G)$ such that $v_1, e_1, v_2, e_2, \dots, v_t, e_t, v_1$ is a walk with at least one edge and $v_1, v_2, \dots, v_t$ are distinct.