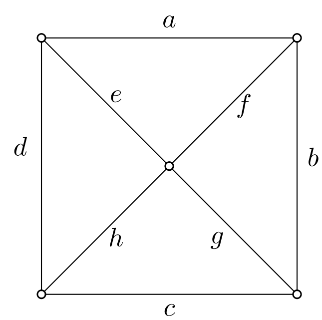
**Total marks available: 60**

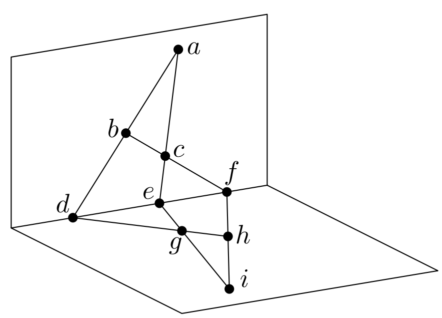
*You may use results stated in the course notes and lecture notes, but you should cite results that you use. You should not use any other resources, printed or online. This is not a collaborative assignment – your submitted work should reflect your individual effort.*

**Q1.** Consider the following graph $G$.




Draw a geometric representation of $M(G)$. Label the elements appropriately. **[4]**

**Q2.** Consider the rank-4 matroid $M$ with the geometric representation given below (and also seen in Assignment 1 Q1). Draw a graph $G$ such that $M = M(G)$. Label the edges of $G$ appropriately.



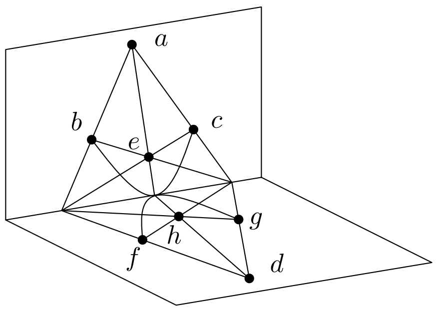

**[4]**

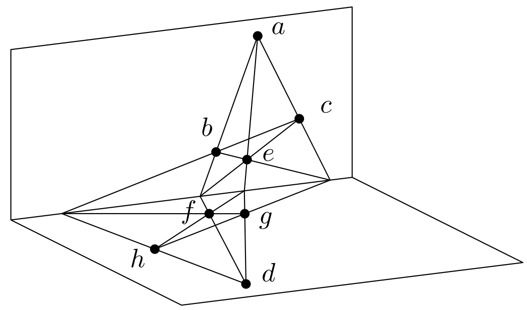
**Q3.** Draw a geometric representation of the matroid that has a representation over GF(3) given by the following matrix. Label the elements, and provide some working.

$$\begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 2 \end{bmatrix}$$

**[6]**

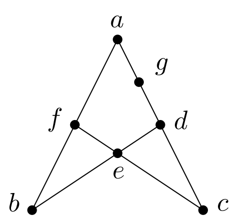
**Q4.** The following diagram shows geometric representations of two rank-4 matroids. Find a matrix that represents the first matroid over the field GF(2), and a matrix that represents the second over GF(3). Label the columns appropriately, and provide some working.




**[10]**

**Q5.** Let $M$ be the rank-3 matroid shown below.




Draw a geometric representation of $M^*$. Show some working. **[6]**

**Q6.** Let $M$ be a matroid, and let $e$ and $f$ be elements of $E(M)$ that are not coloops. Prove that $\{e, f\}$ is a cocircuit of $M$ if and only if every circuit of $M$ that contains one of $e$ and $f$ contains both. **[5]**

**Q7.** A matroid is *self-dual* if it is isomorphic to its dual. Prove that if $M$ is a self-dual matroid with ground set $E$, then $|E|$ is even. **[4]**

**Q8.** Recall *sparse paving* matroids from Assignment 1 Q8. A matroid $M$ is *sparse paving* if and only if every circuit of $M$ has cardinality at least $r(M)$, and whenever $C$ and $C'$ are distinct circuits of $M$ with size $r(M)$, then $|C \cap C'| < r(M) - 1$.[1]

(i) Let $M$ be a sparse paving matroid with rank $r$. Prove that if $C$ is a circuit in $M$ of size $r$, then $C$ is a hyperplane.

(ii) Prove that if $M$ is sparse paving, then $M^*$ is sparse paving. **[9]**

**Q9.** Let $M$ be a matroid on ground set $E$ with $r$ as its rank function. Recall that we use $2^E$ to denote the power set of $E$. Define a new function, $r^*\colon 2^E \to \mathbb{Z}$ by the equation $r^*(X) = r(E - X) + |X| - r(M)$ for every subset $X \subseteq E$. Prove directly that $r^*$ satisfies the three conditions **R1**, **R2**, and **R3**, using only the fact that $r$ satisfies **R1**, **R2**, and **R3**, and no other facts about matroid duality. **[8]**

**Q10.** Find an infinite sequence of graphs $G_4, G_5, G_6, \ldots$ such that $G_i$ has exactly $i$ vertices for each $i$, and $M(G_i)$ has a circuit that is also a hyperplane. **[4]**

[1]You do not need to prove this statement (it follows easily from the definition seen previously).