Quantum Field Theory

Lecture 10

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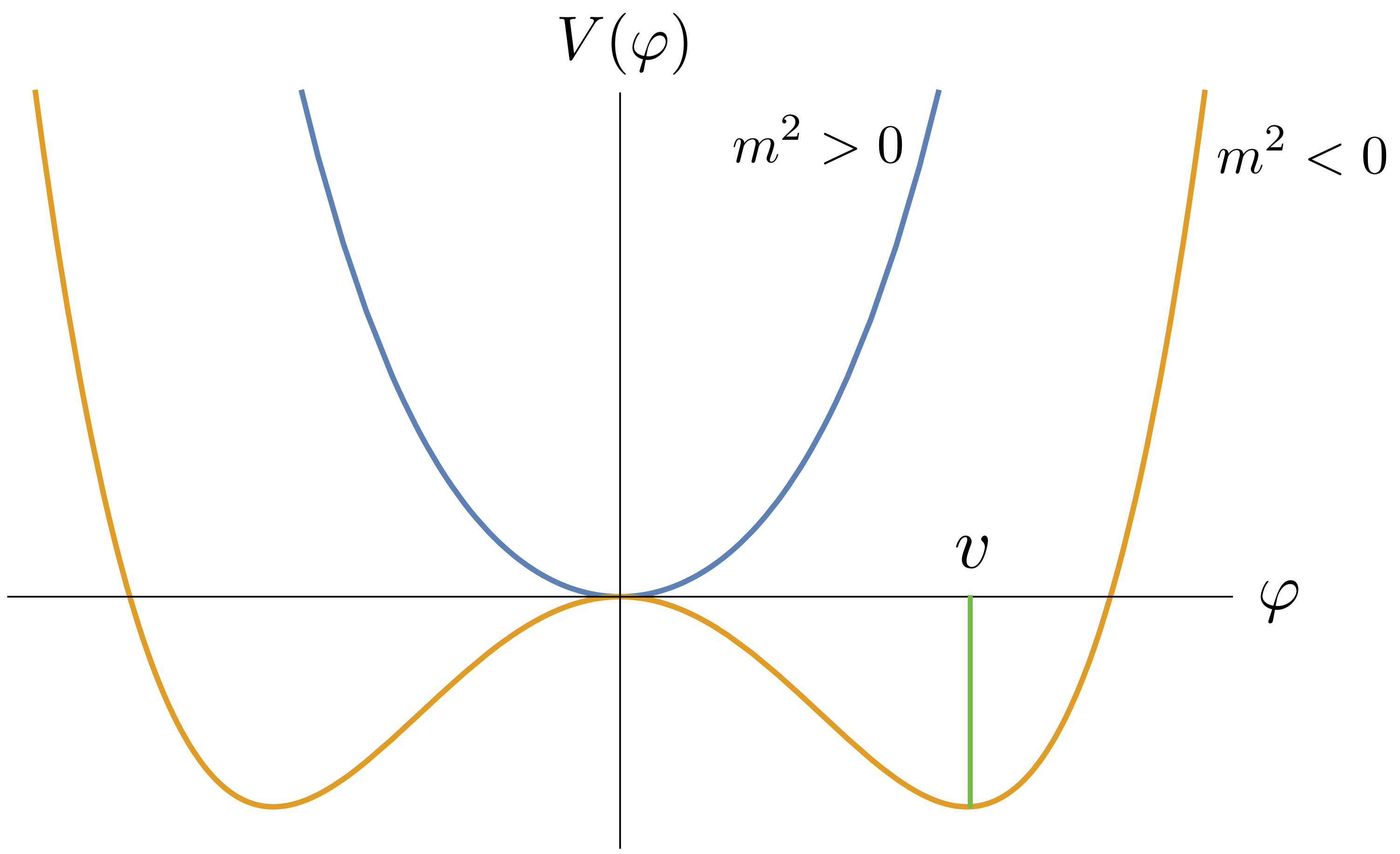
Spontaneous symmetry breaking and the Higgs mechanism

$$\mathcal{L} = -\partial^\mu \varphi^\dagger \partial_\mu \varphi - V(\varphi)$$

$$V(\varphi) = m^2 \varphi^\dagger \varphi + \frac{1}{4}\lambda(\varphi^\dagger \varphi)^2$$



U(1) symmetry

$$\varphi(x) = \frac{1}{\sqrt{2}}[v + a(x) + ib(x)]$$

$$v = (4|m^2|/\lambda)^{1/2}$$

$$\mathcal{L} = -\tfrac{1}{2}\partial^\mu a\partial_\mu a - \tfrac{1}{2}\partial^\mu b\partial_\mu b - V(\varphi)$$

$$V(\varphi) = \tfrac{1}{4}\lambda v^2 a^2 + \tfrac{1}{4}\lambda v a(a^2 + b^2) + \tfrac{1}{16}\lambda(a^2 + b^2)^2$$

$$m_a^2 = \tfrac{1}{2}\lambda v^2 \qquad \underline{m_b^2 = 0}$$

b = massless “Goldstone boson”

U(1) symmetry is “spontaneously broken”

Better parameterization of the fields:

$$\varphi(x) = \frac{1}{\sqrt{2}}[v + h(x)] \exp[i\chi(x)/v]$$

$$\mathcal{L} = -\tfrac{1}{2}\partial^\mu h \partial_\mu h - \tfrac{1}{2}(1 + v^{-1}h)^2 \partial^\mu \chi \partial_\mu \chi - V(\varphi)$$

$$V(\varphi) = \tfrac{1}{4}\lambda v^2 h^2 + \tfrac{1}{4}\lambda v h^3 + \tfrac{1}{16}\lambda h^4$$

$$m_h^2 = \tfrac{1}{2}\lambda v^2 \qquad \underline{m_\chi^2 = 0}$$

Scalar electrodynamics

$$\mathcal{L} = -(D^\mu\varphi)^\dagger D_\mu\varphi - m^2\varphi^\dagger\varphi - \tfrac{1}{4}\lambda(\varphi^\dagger\varphi)^2 - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

Gauge transformation:

$$\varphi(x) \to \exp[-ie\Gamma(x)]\varphi(x)$$

$$\varphi^\dagger(x) \to \exp[+ie\Gamma(x)]\varphi^\dagger(x)$$

$$A^\mu(x) \to A^\mu(x) - \partial^\mu\Gamma(x)$$

$$m^2 < 0$$

$$\varphi(x) = \frac{1}{\sqrt{2}}[v + h(x)] \exp[i\chi(x)/v]$$

$$\varphi(x) \to \exp[-ie\Gamma(x)]\varphi(x)$$

Choose $e\Gamma(x) = \chi(x)/v$

$$\varphi(x) \to \frac{1}{\sqrt{2}}[v + h(x)]$$

$\chi(x)$ has disappeared!

$$-(D^\mu \varphi)^\dagger D_\nu \varphi = -\tfrac{1}{2}\partial^\mu h \partial_\mu h - \tfrac{1}{2}e^2 v^2 (1 + v^{-1}h)^2 A^\mu A_\mu$$

$$M_A = ev$$

Vector field has acquired mass
by "eating" the Goldstone field

This is the "Higgs mechanism"

It is how the $W^\pm$, Z^0 get mass
in the Standard Model

Nonabelian gauge theory:

Consider *N* equal-mass Dirac fields:

$$\mathcal{L} = i\overline{\Psi}_j \gamma^\mu \partial_\mu \Psi_j - m\overline{\Psi}_j \Psi_j$$

Global U(*N*) symmetry:

$$\Psi_j(x) \to U_{jk}\Psi_k(x)$$

$$U^\dagger U = I$$

Consider a *local* U(*N*) transformation:

$$\Psi(x)_j \to U_{jk}(x)\Psi_k(x)$$

Introduce a covariant derivative & gauge field:

$$(D^\mu)_{jk} \equiv \delta_{jk}\partial^\mu - ig(A^\mu)_{jk}$$

Require $[D^\mu \Psi(x)]_j \to U_{jk}(x)[D^\mu \Psi(x)]_k$

Equivalently $D^\mu \to U D^\mu U^\dagger$

$$A^\mu \to U A^\mu U^\dagger + \frac{i}{g} U \partial^\mu U^\dagger$$

Field strength:

$$[D^\mu, D^\nu] = -ig\big(\partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]\big) \equiv -igF^{\mu\nu}$$

$$F^{\mu\nu} \to U F^{\mu\nu} U^\dagger$$

Gauge-invariant lagrangian:

$$\mathcal{L} = -\tfrac{1}{2}\mathrm{Tr}\big(F^{\mu\nu}F_{\mu\nu}\big) + i\overline{\Psi}\not{D}\Psi - m\overline{\Psi}\Psi$$

$$D^\mu = \partial^\mu - igA^\mu$$

Can restrict to SU(N):

$$\det U = 1 \qquad \mathrm{Tr}\, A^\mu = 0$$

Quantum chromodynamics (QCD)
of quarks and gluons:
SU(3) “color” group

Electroweak interactions:
$SU(2) \times U(1) \rightarrow U(1)_{EM}$
$W^{\pm}$, Z^0 gain mass via Higgs mechanism

All together, this is
the Standard Model
of elementary particles

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