QFT School: Lecture 3

2.3 Induced Chern-Simons terms

• consider (2+1)D QED: $Z=\int \mathcal{D}\psi\, \mathcal{D}A^\mu\, e^{-S[\psi,A^\mu]}$

$$S=\int d^3x\,\bar\psi\left[\gamma^\mu(i\partial_\mu+eA_\mu)+m\right]\psi$$

⇒ define effective action for vector potential / gauge field

via $$Z=\int \mathcal{D}A^\mu\, e^{-S_{eff}[A^\mu]}$$

with $$e^{-S_{eff}[A^\mu]} \equiv \int \mathcal{D}\psi\, e^{-S[\psi,A^\mu]}$$

⇒ because $S[\psi,A^\mu]$ is quadratic in $\bar\psi,\psi$: straightforward Gaussian integration yields

$$S_{eff}[A^\mu] = \underbrace{\ln\det}_{\equiv\, \mathrm{Tr}\,\ln}\left(\gamma^\mu(i\partial_\mu+eA_\mu)+m\right)$$

⇒ perturbative expansion in A_μ:

$S_{eff}[A^\mu]$ = const +  +  + ...

find  $\underset{\substack{\uparrow \\ \text{long-wavelength,} \\ \text{large-mass} \\ \text{limit}}}{\equiv} \int d^3x\, \mathcal{L}_{CS}$ with $\kappa = \frac{e^2}{4\pi}\frac{m}{|m|}$

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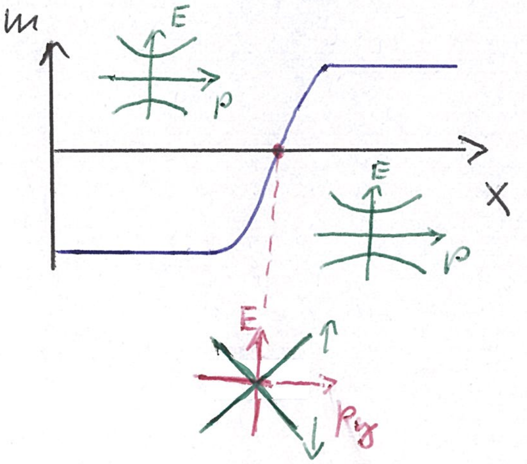
$\Rightarrow \mathcal{H}$ is a topological invariant: independent of magnitude of m and of fixed value as long as $m \neq 0$ (gapped)

distinguish two types of "Dirac insulators" by $m \gtrless 0$, conducting boundary at interface between them!



helical boundary states

Qi et al., PRB 78, 195424 (2008)

- 3D realisation possible in certain materials: topological insulators

$$\mathcal{H} = \begin{pmatrix} m - B\vec{p}^{\,2} & A(p_x + ip_y) & 0 & A' p_z \\ A(p_x - ip_y) & -m + B\vec{p}^{\,2} & -A' p_z & 0 \\ 0 & -A' p_z & m - B\vec{p}^{\,2} & A(p_x - ip_y) \\ A' p_z & 0 & A(p_x + ip_y) & -m + B\vec{p}^{\,2} \end{pmatrix}$$

$m \gtrless 0$ distinguishes normal / topological systems!

$\Rightarrow$ find a topological term for electromagnetism in material:

$$\mathcal{L}_{top} = \frac{1}{2} \frac{P_3(x,t)}{2\pi} \varepsilon^{\mu\nu\sigma\tau} \partial_\mu A_\nu \partial_\sigma A_\tau$$

$\Rightarrow$ consider $S = \int d^4x \left(\mathcal{L}_{top} + j^\mu A_\mu \right)$, find Euler-Lagrange eq.

$$j^\mu = \frac{1}{2\pi} \varepsilon^{\mu\nu\sigma\tau} \partial_\nu P_3 \partial_\sigma A_\tau$$

③

$\Rightarrow$ thus that is only an effect where P_3 changes as function of space and / or time!

more explicitly: $\rho = \frac{1}{2\pi}(\vec{\nabla} P_3) \cdot \vec{B}$

$$\vec{J} = -\frac{1}{2\pi}(\vec{\nabla} P_3) \times \vec{E} - \frac{1}{2\pi}(\partial_t P_3)\vec{B}$$

also rewrite Lagrangian: $\mathcal{L}_{top} = \frac{P_3(x,t)}{2\pi}\,\vec{E}\cdot\vec{B}$

axion electrodynamics!!!

M Franz, Physics 1, 36 (2008)
F Wilczek, PRL 58, 1799 (1987)

$\Rightarrow$ in topological insulators w/ time-reversal symmetry:

$$P_3 = \frac{e^2}{2\pi\hbar}\cdot\pi$$