S

S–CALCF

# Scholarship 2015
# Calculus

2.00 p.m. Tuesday 17 November 2015

## FORMULAE AND TABLES BOOKLET

Refer to this booklet to answer the questions for Scholarship Calculus 93202Q.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

**YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.**

# CALCULUS – USEFUL FORMULAE

## ALGEBRA

### Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b (xy) = \log_b x + \log_b y$

$\log_b \left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b \left(x^n\right) = n \log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

### Complex numbers

$z = x + \mathrm{i}y$

$\quad = r \operatorname{cis}\theta$

$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$\quad = r \operatorname{cis}(-\theta)$

$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

### De Moivre’s Theorem

If $n$ is any integer, then

$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis}(n\theta)$

### Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$$

Some values of $\binom{n}{r}$ are given in the table below.

| $n$ \ $r$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 2 | 1 | | | | | | | | |
| 3 | 1 | 3 | 3 | 1 | | | | | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | | | | | |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 | | | | | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | | |
| 7 | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | | | |
| 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | | |
| 9 | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 | |
| 10 | 1 | 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | 1 |
| 11 | 1 | 11 | 55 | 165 | 330 | 462 | 462 | 330 | 165 | 55 | 11 |
| 12 | 1 | 12 | 66 | 220 | 495 | 792 | 924 | 792 | 495 | 220 | 66 |

## COORDINATE GEOMETRY

### Straight Line

Equation $y - y_1 = m(x - x_1)$

### Circle

$(x-a)^2 + (y-b)^2 = r^2$

has a centre $(a, b)$ and radius $r$

### Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

Focus $(a, 0)$ Directrix $x = -a$

### Ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

Foci $(c, 0)$ $(-c, 0)$ where $b^2 = a^2 - c^2$

Eccentricity: $e = \dfrac{c}{a}$

### Hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\dfrac{b}{a}x$

Foci $(c, 0)$ $(-c, 0)$ where $b^2 = c^2 - a^2$

Eccentricity: $e = \dfrac{c}{a}$

## CALCULUS

### Differentiation

| $y = f(x)$ | $\dfrac{dy}{dx} = f'(x)$ |
|---|---|
| $\ln x$ | $\dfrac{1}{x}$ |
| $e^{ax}$ | $ae^{ax}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |

### Integration

| $f(x)$ | $\int f(x)\mathrm{d}x$ |
|---|---|
| $x^n$ | $\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$ |
| $\dfrac{1}{x}$ | $\ln\lvert x\rvert + c$ |
| $\dfrac{f'(x)}{f(x)}$ | $\ln\lvert f(x)\rvert + c$ |

### First principles

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

### Parametric Function

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

### Product Rule

$(f.g)' = f.g' + g.f'$ or if $y = uv$ then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

### Quotient Rule

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ or if $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

### Composite Function or Chain Rule

$$\left(f(g)\right)' = f'(g).g'$$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

## NUMERICAL METHODS

### Trapezium Rule

$$\int_a^b f(x)\ dx \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + ... + y_{n-1})\right]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

### Simpson's Rule

$$\int_a^b f(x)\ dx \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right]$$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and $n$ is even.

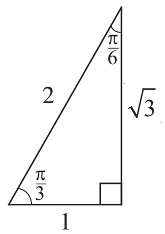
## TRIGONOMETRY

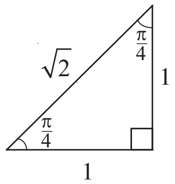
$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$




### Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

### Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

### General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where $n$ is any integer

### Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

### Double Angles

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

### Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

### Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

## MEASUREMENT

### Triangle

Area $= \frac{1}{2}ab\sin C$

### Trapezium

Area $= \frac{1}{2}(a+b)h$

### Sector

Area $= \frac{1}{2}r^2\theta$

Arc length $= r\theta$

### Cylinder

Volume $= \pi r^2 h$

Curved surface area $= 2\pi rh$

### Cone

Volume $= \frac{1}{3}\pi r^2 h$

Curved surface area $= \pi rl$ where $l =$ slant height

### Sphere

Volume $= \frac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

S–CALCF