Question 4 solutions (2015 Q1)

(a)

$$S = 2\pi\int_1^3 \left(x^3 + \frac{1}{12x}\right)\sqrt{1+\left(3x^2 - \frac{1}{12x^2}\right)^2}\,\mathrm{d}x$$

$$= 2\pi\int_1^3 \left(x^3 + \frac{1}{12x}\right)\sqrt{1+9x^4 - \frac{1}{2} + \frac{1}{144x^4}}\,\mathrm{d}x$$

$$= 2\pi\int_1^3 \left(x^3 + \frac{1}{12x}\right)\sqrt{\left(3x^2 + \frac{1}{12x^2}\right)^2}\,\mathrm{d}x$$

$$= 2\pi\int_1^3 \left(x^3 + \frac{1}{12x}\right)\left(3x^2 + \frac{1}{12x^2}\right)\mathrm{d}x$$

$$= 2\pi\int_1^3 \left(3x^5 + \frac{x}{12} + \frac{x}{4} + \frac{1}{144x^3}\right)\mathrm{d}x = 2\pi\int_1^3 \left(3x^5 + \frac{x}{3} + \frac{1}{144x^3}\right)\mathrm{d}x$$

$$S = 2\pi\left[\frac{x^6}{2} + \frac{x^2}{6} - \frac{1}{288x^2}\right]_1^3 = 2295.5$$