Question 6 solutions (2015 Q2)

(a)

$$\left(3^{2x+y}\right)^2 - 3^{2x+y} - 6 = 0$$

$$3^{2x+y} = 3, \quad 3^{2x+y} \neq -2$$

$$2x + y = 1 \Rightarrow y = 1 - 2x$$

Substituting:

$$\log_{x+1}(4 - 2x)(5 - x) = 3$$

$$20 + 2x^2 - 14x = (x+1)^3$$

$$x^3 + x^2 + 17x - 19 = 0$$

$$x = 1$$

$$y = -1$$

(b)   The form of the parabola is $y = kx^2$.
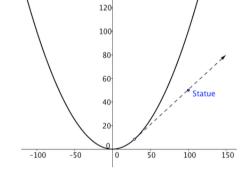
At $(100, 100)$,

$$100 = 100^2 k \Rightarrow k = \frac{1}{100}$$

$$y = \frac{1}{100}x^2$$

Gradient of the tangent at (x_0, y_0):

$$\frac{dy}{dx} = \frac{x_0}{50}$$

$$y_0 = \frac{1}{100}x_0^2$$

Matching gradients: Gradient of tangent: $\dfrac{x_0}{50} = \dfrac{50 - \dfrac{1}{100}x_0^2}{100 - x_0}$ gradient of the line

$$100x_0 - x_0^2 = 2500 - \frac{1}{2}x_0^2$$

$$0 = \frac{1}{2}x_0^2 - 100x_0 + 2500$$

$$x_0 = 29.3$$

Coordinates of the point (29.3, 8.6); 29.3 m east, 8.6 m north.