Memoryless reigns of the ‘Sons of Heaven’: exponential rule lengths revealed and explained

Estate V. Khmaladze

John Haywood

School of Mathematics, Statistics and Operations Research, Victoria University of Wellington, Wellington 6140, New Zealand.

Ray Brownrigg

School of Engineering and Computer Science, Victoria University of Wellington, Wellington 6140, New Zealand.

Summary. Human lifetimes have increasing failure rates; as people age they are more likely to die. Viewing the succession of Chinese Emperors as a statistical ensemble we show that, unlike lifetimes, their reigns ceased at a constant rate for over two millennia, unaffected by elapsed time. So reign lengths of the ‘Sons of Heaven’ followed a memoryless exponential distribution, with a 10-year half-life. Becoming emperor not only influenced the duration of remaining life, but length of rule was independent of age at ascent. We explain the observed exponentiality using asymptotic results from the theory of stochastic processes.

Keywords: Chinese Emperors; constant failure rate; stochastic processes; testing exponentiality.

1. Introduction

In this paper we consider the statistical nature of the durations of rule of Chinese emperors, or the ‘Sons of Heaven’, from 221 BC to 1911 AD; see Paludan (1998) and Table 1. Huge efforts have been made to document and study data of this magnitude, large in both their importance and time spanned. So to view them as a ‘statistical ensemble’, as particles would be considered in statistical physics, may seem simplistic, naive and inadequate. Recall, however, that it could have seemed equally naive and inadequate to study Dante’s “Divina Comedia” or Joyce’s “Ulysses” as statistical ensembles of words. Yet it was precisely such studies which revealed beautiful and unexpected regularities of written texts, known generally under the name of Zipf’s Law (Zipf, 1949; Khmaladze, 2002). Likewise, we show here that statistical analysis of reign lengths of Chinese emperors reveals phenomena which are strange, but very simple and regular.

Human lifetimes have an increasing failure rate – as people age (beyond early childhood) they are more likely to die. The same is true, basically, for all biological organisms: there is always aging, accumulation of faults or damages and, as a result, increasing failure rates. One would think that the same should be true for reigns – a ruler can be removed from power as a result of accumulated controversies and difficulties of political, military, economic or personal nature. This is indeed the case for some European monarchies of modern times (from the 15th century onwards), including the British monarchs; e.g., see Khmaladze *et al.*

E-mail: John.Haywood@vuw.ac.nz

(2007). However, for Chinese emperors it turns out to be completely untrue. Durations of their reign seem in extremely good agreement with an exponential distribution and, hence, have a constant failure rate (Barlow and Proschan, 1975). This implies that any reign would stop as if completely at random, without predictable historical influences and with no 'aging' or accumulation of damages.

Hence, surprisingly, the successions of emperors form one long time-homogeneous Poisson process (Daley and Vere-Jones, 2003), no different in its probabilistic nature to, say, the radioactive emission of α particles first demonstrated by Rutherford a century ago (Rutherford *et al.*, 1910) or the times between non-aftershock earthquakes (Gardner and Knopoff, 1974). The 'half-life' of a Chinese emperors' reign was 10 $\pm$ 1.1 years (estimated median reign $\pm$ s.e.).

In more detail, our statistical analysis reveals the following: ascent to the imperial throne changed the life of an emperor, which no longer followed the usual mechanisms of remaining life (Gerber, 1997); duration of rule, viewed as a random variable, becomes independent of the age at ascent – it is not true, for example, that young emperors ruled for longer; the durations of rule followed the same 'memoryless' exponential distribution, with the same parameter (or failure rate) for more than 21 centuries.

Earlier we observed the same phenomena for Roman emperors (Khmaladze *et al.*, 2007), but initially thought such instability of supreme rule was a statistical manifestation of the 'decline and fall' of the Roman Empire. Imperial China was investigated as a possible counter-example, but instead we have confirmed such occurrences were more widespread. Hence we must conclude even very stable and civilized societies do not require nor imply the stability of any particular ruler. If it is said that bureaucracy ruled China, it did so in a surprisingly consistent and time-homogeneous way.

2. Testing exponentiality

To verify statistically any important hypothesis on data of this magnitude requires appropriate statistical methodology; we used two groups of tests. First we used goodness of fit tests, of a relatively novel form, to verify that the collection of durations of reign agrees with the exponential distribution well. Secondly directed tests were used, to verify that these durations and the ages of ascent are indeed statistically independent random variables. It was also necessary to make small preliminary adjustments (described below) of the data.

The goodness of fit tests we employed were formed using a transformation of the parametric (or estimated) empirical process (Khmaladze, 1981), following the approach for exponentiality detailed recently (Haywood and Khmaladze, 2008). Briefly, for reign length x and a hypothesised exponential distribution $F(x) = 1-\exp(-\lambda x)$ with estimated parameter $\hat{\lambda}$, we centred the empirical distribution function of the reign lengths of n emperors, $F_n(x)$, not by $1-\exp(-\hat{\lambda}x)$ but using the so called compensator $K(x, F_n, \hat{\lambda})$ (Jacod and Shiryaev, 2003). The resulting empirical process,

$$w_n(x) = \sqrt{n}\left\{F_n(x) - K(x, F_n, \hat{\lambda})\right\}$$

converges in distribution to Brownian motion $w(x)$ with $E\{w^2(x)\} = F(x)$. We made our inference on the basis of a two-sided Kolmogorov-Smirnov statistic and an omega-square statistic, evaluated respectively using the limiting distributions (Feller, 1971; Martynov,

1977) of

$$\sup_x \mid w_n(x) \mid \quad \text{and} \quad \int_0^\infty w_n^2(x)dF(x).$$

Recently it was shown that the process w_n converges to its limit quickly and the limit distributions can be confidently used for sample sizes of 50 or bigger (Haywood and Khmaladze, 2008); our sample size is $n = 161$. It has also been shown that tests based on similar processes have relatively good power; e.g., Nikitin (1995). However, the power considerations became almost immaterial here because the agreement with exponentiality turns out to be exceptionally good. For the two-sided Kolmogorov-Smirnov statistic $P = 0.74$ and for the omega-square statistic $P = 0.66$; that is, the statistics are almost perfect exemplars of where they should be under the hypothesis of exponential lengths of rule (see Fig. 1).

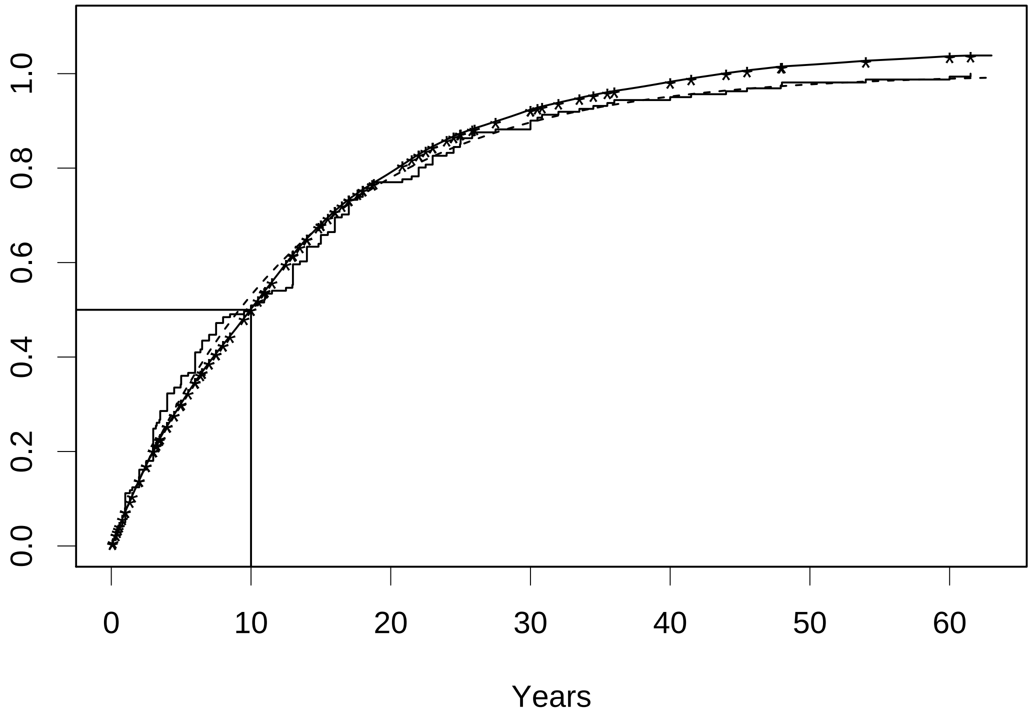

Fig. 1. Empirical distribution function of reign lengths of all ($n = 161$) Chinese emperors (solid step function) with corresponding exponential approximation (dashed line) and compensator (solid line with asterisks). The reign 'half-life' (the median reign length) was estimated as 10 $\pm$ 1.1 years (estimate $\pm$ s.e.), emphasized by the intersecting straight lines.

Next a group of tests was used specifically to verify or reject the following alternative explanation: for humans of age 30-40 years and above, the distribution function of their remaining life has a concave form – e.g., see Khmaladze *et al.* (2007) – as does the exponential distribution function. Hence, the mixture of such remaining life distribution functions over these ages will also have a concave shape. Therefore, the question arises whether the agreement of the empirical distribution function of durations of reign with the exponential distribution is not simply an illusion and whether the durations of reign do not simply follow the distribution of remaining life. This, however, can be tested specifically if we involve in our analysis not only the durations of reign but also the ages at ascent. Indeed, if the latter, simpler, explanation were true, it must have been that older emperors ruled for shorter times than younger ones.

The age of ascent is known for a large subset of the emperors ($n = 131$, Table 1), and we used two statistical tests. For the first of these tests we used Spearman's rank correlation coefficient r_s to check the hypothesis that age at ascent and reign length are uncorrelated against the alternative of a negative correlation. Indeed, the data shows no correlation: $r_s = -0.04$, $P = 0.33$; visually there is a wide spread of reign lengths at all ages of ascent (Fig. 2). Our second test checked the hypothesis of equality of the distribution functions of lengths of rule for young and old Chinese emperors: the emperors were divided into two groups by the median age at ascent of 24 years and empirical distribution functions of their rule lengths were compared using the classical two-sample Kolmogorov-Smirnov statistic (Conover, 1999). The resulting P value was again high ($P = 0.51$; Fig. 3). So there is considerable evidence in the data that there is no association between the age at ascent and the duration of rule and, therefore, those durations do not behave as merely durations of the remaining lives.

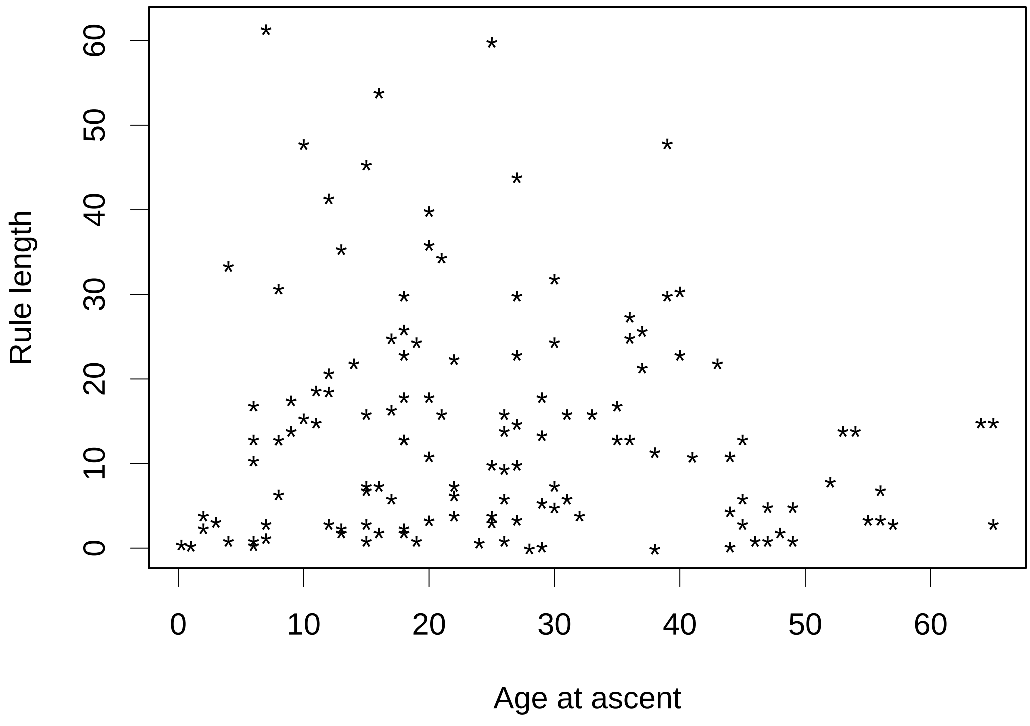

Fig. 2. Reign length by age at ascent for $n = 131$ Chinese emperors (those with known age at ascent). None of these emperors lived beyond 87 years and three were over 60 when their reigns began. However, two of those three reigned for 15 years, which exceeds the average reign of 13 years and two months.

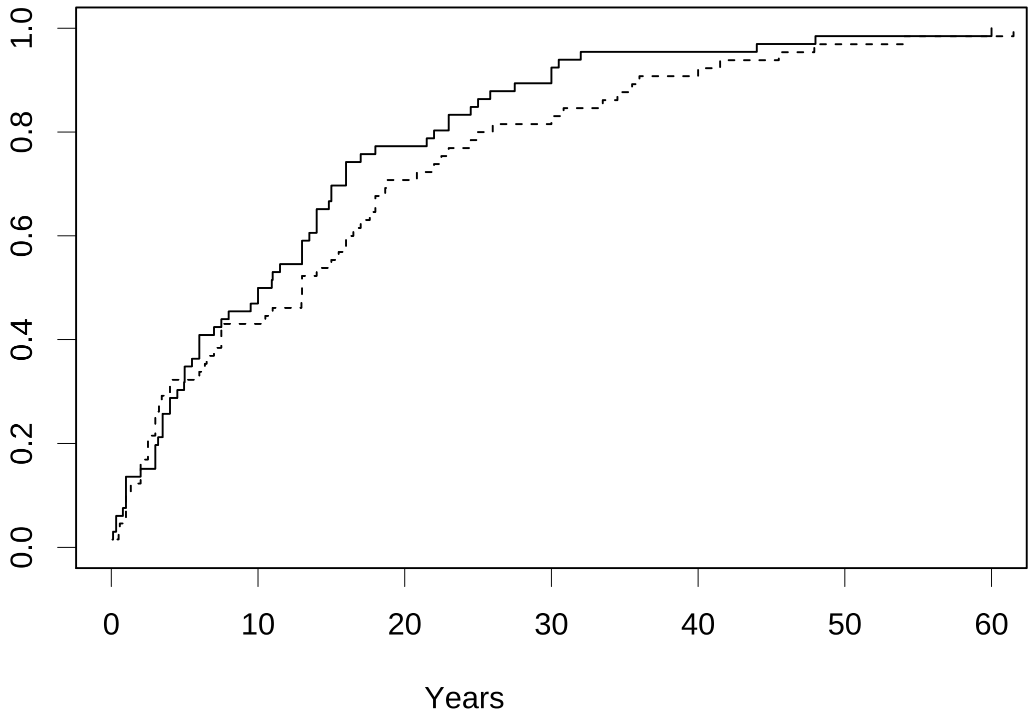

Fig. 3. Comparison of empirical distribution functions of reign lengths for 'young' (dashed line) and 'old' Chinese emperors ($n = 131$); division into age groups by median age at ascent (24 years). It is clear these empirical distributions could come from the same underlying distribution.

2.1. Minor adjustments to reign lengths

The data on reign lengths of Chinese emperors were for those emperors whose rule over China is officially recognised (Paludan, 1998), rather than including all contemporaneous reigns over sub-regions; e.g., see Encyclopaedia Britannica (2002). Paludan (1998) presented the year of ascent and death (or overthrow or abdication) of each officially recognised emperor, from 221 BC to 1911 AD. However, in 14 cases ascent and end-of-reign occurred in the same year, so a simple subtraction of the two values would produce a reign of length zero (see Table 1). Further, while the elapsed time spanned by the data is 2132 years, the sum of reign lengths (end year minus start year for each reign) totals only 2051 years. There were some interregnum periods, but these were generally fairly short and certainly insufficient to explain the discrepancy of 81 years. Hence prior to analysis, some adjustments were necessary to give more accurate reign lengths.

In Table 1 several reign length values do not match the difference between the ascent and reign-end dates, since a more exact reign length was used directly from the source text (Paludan, 1998). For example, "The first reign of Li Zhe, the emperor Zhongzong, lasted for just six weeks in 684" (p. 97). Other than such exact figures, we adjusted the dates as follows, so that the total of the reign lengths was much closer to the time spanned by those reigns. Any reign not sharing a particular year with the preceding or succeeding reign was allocated that whole year. If a particular year was split between two or more reigns it was allocated between them equally, except for those reigns known accurately, where the exact length of time was used. Some of the shorter interregnum periods are specified accurately too (Paludan, 1998), so these were taken into account when making the adjustments. This gave much improved agreement between the elapsed time and the total sum of the reigns (2126.4 years).

3. Exponentiality explained

The fact that the consecutive rule times of Chinese emperors, statistically speaking, form a simple time-homogeneous Poisson process, no matter how thoroughly tested, may be hard to accept unless one obtains a general probabilistic explanation for it. We believe such an explanation can be found within the theory of stationary processes.

One can think about the stresses and challenges on the position of an emperor as a stationary random process. We assume that in their exalted position, emperors, unlike common people, had the ability to overcome and remain unaffected by most of these challenges. Only exceptionally high challenges could lead to the replacement of the emperor. However, in the theory of stationary random processes it is known that the times between exceedances of a high level are independent exponential random variables and thus form a time-homogeneous Poisson process (Cramér and Leadbetter, 1967). Two circumstances will prevent the application of the same reasoning to people in more common states of life. First, the process of stresses on more ordinary positions is much less intensive and second, even moderate and weak stresses incur some damages and lives stop not only as the result of a one time high stress but as the result of the accumulation of such damages – the process of aging takes place.

So donning the mantle of ‘Son of Heaven’ was indeed a life-changing phase transition. In particular, the distribution of the duration of remaining life was altered, such that reign lengths did not age in the way human lifetimes do. Instead they followed a simple stochastic mechanism, integral to many processes studied in the natural and physical sciences.

Table 1: Chronology of Chinese emperors, 221 BC to 1911 AD. The last column shows minor adjustments to reign length, described in the text.

Name	Born (year)	Ascent (year)	Reign end (year)	Died (year)	Reign length (years)	Ascent age (years)	Adjusted reign (years)
Qin Shihuangdi	-259	-221	-210	-210	11	38	11.5
Er Shi	-230	-210	-207	-207	3	20	3.44
Gaodi	-247	-206	-195	-195	11	41	10.94
Huidi	-210	-195	-188	-188	7	15	7.5
Wendi	-202	-180	-157	-157	23	22	22.5
Jingdi	-188	-157	-141	-141	16	31	16
Wudi	-157	-141	-87	-87	54	16	54
Zhaodi	-95	-87	-74	-74	13	8	12.96
Xuandi	-91	-74	-49	-49	25	17	24.96
Yuandi	-75	-49	-33	-33	16	26	16
Chengdi	-51	-33	-7	-7	26	18	26

continued . . .

Name	Born (year)	Ascent (year)	Reign end (year)	Died (year)	Reign length (years)	Ascent age (years)	Adjusted reign (years)
Aidi	-26	-7	-1	-1	6	17	6
Pingdi	-9	-1	6	6	7	8	6.5
Ruzi	5	7	9	25	2	2	2.5
Wang Mang	-45	9	23	23	14	54	14
Guang Wudi	-5	25	57	57	32	30	32
Mingdi	28	57	75	75	18	29	18
Zhangdi	57	75	88	88	13	18	13
Hedi	79	88	106	106	18	9	17.62
Shangdi	105	106	106	106	0.58	0.25	0.58
Andi	94	106	125	125	19	12	18.67
Shundi	115	125	144	144	19	11	18.79
Chongdi	143	144	145	145	0.42	1	0.42
Zhidi	138	145	146	146	1.33	7	1.33
Huandi	132	146	168	168	22	14	22
Lingdi	156	168	189	189	21	12	20.83
Xiandi	181	189	220	234	31	8	30.83
Wendi	NA	220	226	226	6	NA	6.5
Mingdi	NA	227	239	239	12	NA	13
Shaodi	NA	240	253	274	13	NA	14
Gao Gui Xiang Gong	NA	254	260	260	6	NA	6.5
Yuandi	NA	260	264	303	4	NA	4.5
Wudi	NA	265	289	289	24	NA	25
Huidi	NA	290	306	306	16	NA	17
Huaidi	NA	307	312	312	5	NA	6
Mindi	NA	313	316	316	3	NA	4
Yuandi	NA	317	322	322	5	NA	6
Mingdi	NA	323	325	325	2	NA	3
Chengdi	NA	326	342	342	16	NA	17
Kangdi	NA	343	344	344	1	NA	2
Mudi	NA	345	361	361	16	NA	17
Aidi	NA	362	365	365	3	NA	4
Hai Xi Gong	NA	366	370	370	4	NA	5
Jian Wendi	NA	371	372	372	1	NA	2
Xiao Wudi	NA	373	396	396	23	NA	24
Andi	NA	397	418	418	21	NA	22
Gongdi	NA	419	419	419	0	NA	1
Wudi	355	420	422	422	2	65	3
Ying Yang Wang	404	423	423	424	0	19	1
Wendi	406	424	453	453	29	18	30
Xiao Wudi	448	454	464	464	10	6	10.5
Mingdi	449	465	472	472	7	16	7.5
Cang Wu Wang	471	473	476	477	3	2	4
Shundi	464	477	479	479	2	13	2.5

continued . . .

Name	Born (year)	Ascent (year)	Reign end (year)	Died (year)	Reign length (years)	Ascent age (years)	Adjusted reign (years)
Gaodi	423	479	482	482	3	56	3.5
Wudi	439	483	493	493	10	44	11
Mingdi	447	494	498	498	4	47	5
Dong Hunhou	481	499	500	501	1	18	2
Hedi	486	501	501	501	0	15	1
Wudi	463	502	549	549	47	39	48
Jian Wendi	501	550	550	550	0	49	1
Yu Zhang Wang	505	551	551	551	0	46	1
Yuandi	507	552	554	554	2	45	3
Jingdi	542	555	556	558	1	13	2
Wudi	500	557	559	559	2	57	3
Wendi	504	560	566	566	6	56	7
Lin Hai Wang	551	567	568	570	1	16	2
Xuandi	516	569	582	582	13	53	14
Hou Zhu	552	583	589	589	6	31	6
Wendi	541	581	604	604	23	40	23
Yangdi	569	604	617	617	13	35	13
Gongdi	611	617	618	618	1	6	1
Gaozu	566	618	626	635	8	52	8
Taizong	599	626	649	649	23	27	23
Gaozong	628	649	683	683	34	21	34.5
Zhongzong	656	684	684	710	0.12	28	0.12
Ruizong	662	684	690	716	6	22	6.38
Wu Zetian	625	690	705	705	15	65	15
Zhongzong	656	705	710	710	5	49	5
Ruizong	662	710	712	716	2	48	2
Xuanzong	685	712	756	762	44	27	44
Suzong	711	756	762	762	6	45	6
Daizong	727	762	779	779	17	35	17
Dezong	742	779	805	805	26	37	25.83
Shunzong	761	805	805	806	0	44	0.33
Xianzong	778	805	820	820	15	27	14.83
Muzong	795	820	824	824	4	25	4
Jingzong	809	824	827	827	3	15	3
Wenzong	809	827	840	840	13	18	13
Wuzong	814	840	846	846	6	26	6
Xuanzong	810	846	859	859	13	36	13
Yizong	833	859	873	873	14	26	14
Xizong	862	873	888	888	15	11	15
Zhaozong	867	888	904	904	16	21	16
Aidi	892	904	907	908	3	12	3
Taizu	852	907	910	910	3	55	3.5
Modi	NA	911	923	923	12	NA	12.5

continued . . .

Name	Born (year)	Ascent (year)	Reign end (year)	Died (year)	Reign length (years)	Ascent age (years)	Adjusted reign (years)
Zhuangzong	NA	923	926	926	3	NA	3
Mingzong	NA	926	934	934	8	NA	8
Feidi	NA	934	935	935	1	NA	1.5
Gaozu	NA	936	944	944	8	NA	8.5
Chudi	NA	944	947	947	3	NA	3
Gaozu	NA	947	948	948	1	NA	1
Yindi	NA	948	951	951	3	NA	3
Taizu	NA	951	954	954	3	NA	3
Shizong	NA	954	960	960	6	NA	6
Taizu	927	960	976	976	16	33	16
Taizong	939	976	997	997	21	37	21.5
Zhenzong	968	998	1022	1022	24	30	24.5
Renzong	1010	1022	1063	1063	41	12	41.5
Yingzong	1032	1064	1067	1067	3	32	4
Shenzong	1048	1068	1085	1085	17	20	18
Zhezong	1076	1086	1101	1101	15	10	15.5
Huizong	1082	1101	1125	1135	24	19	24.5
Qinzong	1100	1126	1126	1156	0	26	1
Gaozong	1107	1127	1162	1187	35	20	36
Xiaozong	1127	1163	1190	1194	27	36	27.5
Guangzong	1146	1190	1194	1200	4	44	4.5
Ningzong	1168	1195	1224	1224	29	27	30
Lizong	1205	1225	1264	1264	39	20	40
Duzong	1240	1265	1274	1274	9	25	10
Gongzong	1270	1275	1275	1323	0	4	1
Duanzong	1269	1276	1278	1278	2	7	3
Bing Di	1272	1279	1279	1279	0	6	0.5
Khubilai	1215	1279	1294	1294	15	64	15
Temur Oljeitu	1265	1294	1307	1307	13	29	13.5
Khaishan	1281	1308	1311	1311	3	27	3.5
Ayurbarwada	1285	1311	1320	1320	9	26	9.5
Shidebala	1303	1321	1323	1323	2	18	2.5
Yesun Temur	1293	1323	1328	1328	5	30	4.96
Tugh Temur	1304	1328	1329	1332	1	24	0.79
Khoshila	1300	1329	1329	1329	0	29	0.33
Tugh Temur	1304	1329	1332	1332	3	25	3.19
Toghon Temur	1320	1333	1368	1370	35	13	35.5
Hongwu	1328	1368	1398	1398	30	40	30.5
Jianwen	1377	1399	1402	1402	3	22	4
Yongle	1360	1403	1424	1424	21	43	22
Hongxi	1378	1425	1425	1425	0	47	1
Xuande	1399	1426	1435	1435	9	27	10
Zhengtong	1427	1436	1449	1464	13	9	14

continued . . .

Name	Born (year)	Ascent (year)	Reign end (year)	Died (year)	Reign length (years)	Ascent age (years)	Adjusted reign (years)
Jingtai	1428	1450	1457	1457	7	22	7.5
Tianshun	1427	1457	1464	1464	7	30	7.5
Chenghua	1447	1465	1487	1487	22	18	23
Hongzhi	1470	1488	1505	1505	17	18	18
Zhengde	1491	1506	1521	1521	15	15	16
Jiajing	1507	1522	1567	1567	45	15	45.5
Longqing	1537	1567	1572	1572	5	29	5.5
Wanli	1563	1573	1620	1620	47	10	47.92
Taichang	1582	1620	1620	1620	0.08	38	0.08
Tianqi	1605	1621	1627	1627	6	15	7
Chongzhen	1611	1628	1644	1644	16	17	16.5
Shunzhi	1638	1644	1661	1661	17	6	17
Kangxi	1654	1661	1722	1722	61	7	61.5
Yongzheng	1678	1723	1735	1735	12	45	13
Qianlong	1711	1736	1795	1795	59	25	60
Jiajing	1760	1796	1820	1820	24	36	25
Daoguang	1782	1821	1850	1850	29	39	30
Xianfeng	1831	1851	1861	1861	10	20	11
Tongzhi	1856	1862	1874	1874	12	6	13
Guangxu	1871	1875	1908	1908	33	4	33.5
Puyi	1905	1908	1911	1967	3	3	3.25

References

Barlow, R. E. and Proschan, F. (1975) *Statistical Theory of Reliability and Life Testing: Probability Models.* New York: Holt, Rinehart and Winston.

Conover, W. J. (1999) *Practical Nonparametric Statistics*, 3rd edn. New York: Wiley.

Cramér, H. and Leadbetter, M. R. (1967) *Stationary and Related Stochastic Processes: Sample Function Properties and Their Applications.* New York: Wiley.

Daley, D. J. and Vere-Jones, D. (2003) *An Introduction to the Theory of Point Processes, Vol. 1: Elementary Theory and Methods*, 2nd edn. New York: Springer.

Encyclopaedia Britannica (2002) China. In *Encyclopaedia Britannica, vol. 3*, p. 222. Chicago: Encyclopaedia Britannica.

Feller, W. (1971) *An Introduction to Probability Theory and its Applications, Volume II*, 2nd edn. New York: Wiley.

Gardner, J. K. and Knopoff, L. (1974) Is the sequence of earthquakes in Southern California, with aftershocks removed, Poissonian? *Bull. Seismol. Soc. Am.*, **64**, 1363–1367.

Gerber, H. U. (1997) *Life Insurance Mathematics*, 3rd edn. Berlin: Springer.

Haywood, J. and Khmaladze, E. V. (2008) On distribution-free goodness-of-fit testing of exponentiality. *J. Econometrics*, **143**, 5–18.

Jacod, J. and Shiryaev, A. (2003) *Limit Theorems for Stochastic Processes*, 2nd edn. Berlin: Springer.

Khmaladze, E. V. (1981) Martingale approach in the theory of goodness of fit tests. *Theor. Prob. App.*, **26**, 240–257 .

Khmaladze, E. V. (2002) Zipf's Law. In *Encyclopaedia of Mathematics, Supplement III* (ed M. Hazewinkel), pp. 460–463. Dordrecht: Kluwer Academic Publishers.

Khmaladze, E. V., Brownrigg, R. and Haywood, J. (2007) Brittle power: On Roman Emperors and exponential lengths of rule. *Stat. Prob. Lett.*, **77**, 1248–1257.

Martynov, G. (1977) *Omega Square (Cramer–von Mises) Test.* Moscow: Nauka (in Russian).

Nikitin, Y. (1995) *Asymptotic Efficiency of Nonparametric Tests.* Cambridge: Cambridge University Press.

Paludan, A. (1998) *Chronicle of the Chinese Emperors: The Reign by Reign Record of the Rulers of Imperial China.* New York: Thames and Hudson.

Rutherford, E., Geiger, H. and Bateman, H. (1910) The probability variations in the distribution of α particles. *Philos. Mag.*, **20**, 698–707.

Zipf, G. K. (1949) *Human Behaviour and the Principles of Least Effort.* Reading, MA: Addison-Wesley.