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Abstracts

Algorithmic Randomness and Derandomization

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Kolmogorov complexity is a tool to measure the information content of strings. Strings with high Kolmogorov complexity are said to be "K-random". The study of this notion of randomness has a long history and it has close connections with the theory of computability. The set of K-random strings has long been known to be undecidable.

Derandomization is a fairly recent topic in complexity theory, providing techniques whereby probabilistic algorithms can be simulated efficiently by deterministic algorithms.

In this talk, I will present some new and surprising (or bizarre?) connections between these fields. In particular, we will show that everything PSPACE is poly-time reducible to the set of K-random strings, and we investigate the question of whether or not $PSPACE$ is PRECISELY the set of decidable sets poly-time reducible to the K-random strings.

Some of this material is from the FOCS 2002 paper "Power from Random Strings" and some is from a more recent paper "What is Efficiently Reducible to the K-Random Strings".

Weakly Bounded Banach Lie Groups

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Some results on the geometrical and algebraic structure of Banach Lie groups satisfying weak compactness conditions.

When All is Said and Done, How should we Play and What should we Expect?

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Modern game theory was born in 1928, when John von Neumann published his Minimax Theorem. This theorem ascribes to all two-person zero-sum games a value — what rational players may expect — and optimal strategies — how they should play to achieve that expectation. Seventy-five years later, strategic game theory has not gotten beyond that initial point, insofar as the basic questions of value and optimal strategies are concerned. To be sure, we do have equilibrium theories; but when the game is not two-person zero-sum, these theories do not provide answers to the above questions. Here, we return to square one: abandon all ideas of equilibrium and simply ask, how should a rational player play, and what should he expect. We provide answers to both questions, for all n-person games in strategic ("normal") form.

Cuntz-Krieger Algebras of Directed Graphs and Shift Spaces

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A directed graph $E = (E^0, E^1, r, s)$ consists of countable sets E^0 of vertices, and E^1 of edges, together with maps $s : E^1 \to E^0$ and $r : E^1 \to E^0$ describing where the edges begin and end. Alex Kumjian, David Pask and Iain Raeburn have defined a universal C^*-algebra $C^*(E)$ associated to the directed graph E. These graph C^*-algebras form an important class of examples of C^*-algebras.

In this talk we give an overview of some of the relationships between the properties of the C^*-algebras of directed graphs and the properties of shift spaces that are naturally associated to these graphs.

Joint Minimization with Alternating Bregman Proximity Operators

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We investigate two notions of proximity operators associated with Bregman distances. These operators are shown to inherit key properties of the standard Moreau proximity operator. Through this analysis, we establish the convergence of an alternating minimization procedure for solving a class of joint optimization problems. Various applications are provided.

Implicit Smoothing of Radial Basis Functions

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A radial basis function (RBF) s is a function of the form

$$s(\bullet) = p(\bullet) + \sum_{i=1}^{N} \lambda_i \Phi(\bullet - x_i)$$

where p is a low degree polynomial and Φ is some (usually radial) function. The energy minimisation characterisations of an important class of RBFs, called polyharmonic splines, make them ideally suited to many scattered data fitting problems. Further, another class of RBFs, the compactly supported RBFs, also have a place in any toolkit for scattered data interpolation.

The introductory part of the talk will outline some of the basic properties of RBFs and some of the characteristics of recent applications of them. This part of the talk will hide some of the detail of the underlying mathematics, preferring instead to illustrate concepts through animations and videos.

The latter part of the talk will concentrate on a smoothing technique for RBFs called implicit smoothing. Theory for this method has recently been developed by the authors. The mollification formulas on which the technique is based can be derived via a mix of techniques from special functions and distribution theory. Some aspects of this derivation will be discussed.

Prevention of the Secession: Hedonic Setting

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We consider the variant of coalition formation model, where a society is divided into groups, and each group subsequently chooses a public project from the unidimensional set, over which individuals have single-peaked preferences, as well as the way to finance this project. Project cost is constant and independent on the nature of the group. We compare the TU-model of this cooperative game with the hedonic model, when the choice of each coalition could be predicted in advance, and thus an agent can form preference ordering over coalitions to be in, and argue that severe ex-ante restrictions on the negotiation possibilities, reducing choice to a singleton, lead to instability, both in the cooperative (core) and non-cooperative (Nash equilibrium in pure strategies) sense. We also discuss stability implications of less severe restrictions on choice, such as different normative equity requirements.

Metrics Associated to Multivariate Polynomial Inequalities

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We discuss distances of Caratheodory type (as introduced by Dubiner) and Finsler type that are determined by multivariate polynomial inequalities of Markov/Bernstein type.

Sarason's Conjecture for Harmonic Polynomials

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A harmonic polynomial f of degree n is the sum of a complex polynomial of degree n and the conjugate of another polynomial of degree m where $m < n$. It is well known that such a polynomial has at most n^2 zeroes. Lately, a conjecture of Sheil-Small stating that if $m = 1$ then the polynomial has at most $3n - 2$ zeroes was proved by Khavinson and Swiatek. Whether this is sharp or not is still open for $n > 4$. Sarason conjectured that there exists a polynomial $p(z)$ with the property that every root of $p'(z) = 0$ is also a root of $p(z) = \overline{z}$. If true, this conjecture in particular proves the sharpness of Sheil-Small conjecture. In this talk we prove Sarason's conjecture for $n =$ 5,6 and 8.

The Cayley Transform and Uniformly Bounded Representations

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The Cayley transformation between $\mathbf{R}^n$ and the sphere S^n induces mappings T_z from spaces of functions on S^n to spaces of functions on $\mathbf{R}^n$ by composition followed by multiplication by the z^{th} power of the Jacobian. This map is trivially L^p-bounded (when $p \operatorname{Re} z = 1$), and it follows from this that $\mathrm{SO}(n, 1)$ acts isometrically on certain L^p-spaces. The complementary series of unitary representations arises because T_z is also an isometry between a certain Sobolev-type space on S^n and the usual homogeneous Sobolev space on $\mathbf{R}^n$. The aim of this talk is to review this, and to extend the results to Cayley transformations between the Iwasawa N and K/M for other simple Lie groups of real rank one.

Entropy for Amenable and Non-singular Actions

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I will describe some recent work with Golodets on the study of measure-preserving amenable actions which have completely positive entropy.

I will also describe some results with Genevieve Mortiss on a possible approach to defining entropy for non-singular actions.

Hard Problems and Parameterized Reductions

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There has been a lot of interest recently on exact algorithms for hard problems, subexpenential time, and realted issues such as polynomial time approximation algorithms. We will show how parameterized complexity can shed considerable light on these issues. This talk will be based on the paper delivered last year to the annual conference on computational complexity "Parameterized Complexity for the Skeptic", which can be found on my homepage.

Algorithmic Randomness and Complexity

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I wish to report on joint work [3, 1, 2, 4] looking at the Kolmogorov complexity of reals, and calibrating the relative randomness on reals. Additionally, I will report on some remarkable results due to Miller, Stephan, Nies, Yu Liang and others related to this material.

The basic idea is to try to calibrate the leven of randomness of a real by looking at things like relative initial segment complexity, and to try to understant how relative randomness relates to things like relative computability. There has been intensive activity in this area in the last 3 years. A monograph [5] is in preparation. For instance, in [1], with Hirschfeldt, Nies and Stephan, we analysed the collection of "trivial" reals, namely those whose prefix-free Kolmogorov complexity was the same as $\mathbb{N}$. Such reals provide an easy injury-free solution to Post's problem. Recently, Miller and Yu have shown that initial segment complexity is enough to characterize n-randomness for all n. This supports the thesis of Stephan that unless reals compute $\mathbf{0}'$, then they are computationally very weak. I will also mention other varieties of randomness. For instance, in [2], with Griffiths, I looked at Schnorr randomness which can be argued is a better notion of randomness from the point of view of effective statistical tests. We obtained a machine characterization of this notion for the very first time.

1. R. Downey, D. Hirschfeldt, A. Nies and F. Stephan, *Trivial Reals*, in: *Proceedings of the 7th and 8th Asian Logic Conferences*, World Scientific, 2003, 103–131.

2. R. Downey and E. Griffiths, *Schnorr Randomness*, Jour. Symbolic Logic, to appear.

3. R. Downey, D. Hirschfeldt, A. Nies, and S. Terwijn, *Calibrating randomness,* Bull. Symbolic Logic, to appear.

4. R. Downey, E. Griffiths, D. Hirschfeldt, and A. Nies, *On* Ω^A, in preparation.

5. R. Downey, and D. Hirschfeldt, *Algorithmic Randomness and Complexity*, monograph in preparation, Springer-Verlag. Preliminary version can be found at `http://www.mcs.vuw.ac.nz/~downey`.

Function Spaces and Metrisability of Manifolds

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At `http://www.math.auckland.ac.nz/~gauld/research/metrisability.pdf` I have a collection of nearly 100 conditions equivalent to metrisability for a topological manifold. These conditions range widely, from the manifold being properly embeddable in Euclidean space through the tangent microbundle being equivalent to a fibre bundle to the manifold satisfying what in a general topological space is a rather weak covering property. In this talk I shall discuss some of the conditions one may impose on a function space which yield metrisability for a manifold. For example metrisability of a manifold M is equivalent to each of the properties second countable and the function space $C_k(M, \mathbb{R})$ is second countable. Consequently all metrisable manifolds are separable but the converse turns out to be false. On the other hand separability of $C_k(M, \mathbb{R})$ is equivalent to metrisability of M.

Some Developments on Orbit Equivalence

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Both in the measurable and in the Borel case, hyperfinite actions are well understood and classified up to orbit equivalence. In particular any free Borel action of Z^2 is (orbit equivalent to) hyperfinite. In the case of (minimal) topological actions of Z^2 on the Cantor set, the same question is still open. In this talk, I will present recent developments in the study of this problem. These developments come from a work in progress with I. Putnam (Victoria) and C. Skau (Trondheim).

An Efficient Recognition of Digital Surface in Z^n

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One of the efficient recognition(approximation) method of digital surface in Z^n is introduced, where Z means the set of integers. And further, the digital topological properties of digital surfaces are investigated.

Spectral Properties of the Triangle Groups

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By considering a conformal map from the fundamental domain of a triangle group to the plane and studying the pull back of the Laplace-Beltrami operator we can find information about the spectral properties of the triangle groups.

On the Sparse Grid Approximation in Radial Basis Function Spaces — an Application of the Concentration Function

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The concentration function introduced by Gromov and Milman in [A topological application of the isoperimetric inequality, Amer.J.Math., 1983,Vol. 105, 843–854] has been used by Talagrand in [A new look at independence, The Annals of Probability, 1996, Vol.24, No.1, 1–34] to show the concentration of function values of Lipshitz-continuous functions.

This result can be interpreted as an approximation theorem showing how well continuous functions are approximated by constants. In [Additive Models in High Dimensions, Research Report 99–33, SMCS, Victoria University of Wellington], Hegland and Pestov have shown how this result can be extended to derive approximations for additive functions.

These earlier results are extended and specific bounds for the approximation by sparse grid functions are given when the underlying functions are defined by linear combinations of radial basis functions. The discussion is related to multipole expansions and special cases of exponential and Gaussian radial basis functions are provided. The approximations are applied to the case of product spaces with Hamming and Euclidean metrics are discussed.

Radial basis functions are an important tool in modern machine learning [F. Cucker and S. Smale. On the mathematical foundations of learning. Bulletin of AMS, 39:1–49, 2001] in regularisation networks and recent results have shown that sparse grids can provide predictive models which are competitive for data mining applications [Data mining with sparse grids, 2001, J. Garcke, M. Griebel, M. Thess Computing 67, p. 225–253], [High Dimensional Smoothing Based on Multilevel Analysis, Markus Hegland, Ole Moeller Nielsen, Zuowei Shen, 1999, http://datamining.anu.edu.au/publications.html]. Efficient parallel algorithms for sparse grid fitting have been discussed in [Additive Sparse Grid Fitting, M. Hegland, Curve and Surface Fitting: St Malo 2002, p. 209–218, 2002].

The following discussion demonstrates how sparse grids can be as effective as radial basis functions for high dimensional function approximation.

The Structural Properties of Graph C*-algebras

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We discuss recent progress in the investigations of the class of generalized Cuntz-Krieger C*-algebras related to infinite directed graphs. In particular, we present results on the ideal structure and the real and stable ranks of these algebras.

Two Subfactors

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The study of subfactors can be thought of as a "quantization" of the study of subspaces of Hilbert Space. The most general situation of two subspaces is completely understood. We will present some results in the direction of understanding the general situation of two subfactors.

The Langlands Program

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I will outline the Langlands program and describe some of the recent achievements with the emphasis on the geometric approach.

Interactions between Operator Theory in Krein spaces, Linear Fractional Relations on Operator Balls and Functional Abel-Schroder Type Equations

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The Koenigs embedding problem for a semigroup of iterates of a holomorphic mapping in Banach space is considered in relations to Abel-Schroder type functional equations and to operator theory in indefinite metric Krein spaces.

Local Poisson Processes in the Neighbourhood of Convex Body

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Consider a fixed convex body K_0 in R^d and a set K from a "neighbourhood" $\mathcal{K}_\varepsilon$ of K_0: in particular, the Lebesgue measure $\mu(K \triangle K_0)$ for all $K \in \mathcal{K}_\varepsilon$ is small. Let Φ_n be Poisson process in R^d with intensity n. We consider the point process formed by the differences

$$\Phi_n(K) - \Phi_n(K_0) = \Phi_n(K \setminus K_0) - \Phi_n(K_0 \setminus K) \quad (1)$$

when the magnitude of $\mu(K \triangle K_0)$ for all $K \in \mathcal{K}_\varepsilon$ is of order $\varepsilon \sim 1/n$ while $n \to \infty$.

The first question to be answered is where does the limiting process of (1) live.

As to the next questions, recall that using the local Steiner formula the Lebesgue measure of $K \setminus K_0$ and $K_0 \setminus K$ can be expressed as a polynomial in ε with the coefficients being so called support measures of K_0. These measures live on the normal bundle $Nor(K_0)$ of the convex body and characterise it completely. Hence, our second and third questions are how the geometry of K_0 (or its support measures) interfere with the limit theorems for the process (1) and how one can obtain sub-point processes driven by (with the intensity measures equal to) each of the support measures.

1. E. Khmaladze and W. Weil *Local point processes in the neighbourhood of a convex body*, 2004, 26pp, under submission,

2. R. Schneider, *Convex geometry*, Cambridge University Press, 1998.

An Isomorphic Version of the Slicing Problem

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A fundamental question in asymptotic convex geometry asks whether the isotropic constant of an arbitrary convex body in an arbitrary dimension is universally bounded. This is known as the slicing problem. Here we deal with a relaxation of this problem, which asks whether the conjecture holds, up to some universal isomorphism. More explicitely, we ask whether there exist two universal constants A and B, such that for every convex body in any dimension, there exists another body whose isotropic constant is bounded by A, and whose distance to the original body is bounded by B. In this talk we study this question, and show that the answer is affirmative, up to at most a logarithmic factor.

Intertemporal Competition for Water Levels

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Competition for water levels among different economic agents is a rather complex environmental economics problem. It is a game, because the agents are likely to act competitively. It is a dynamic game because water can be accumulated from season to season and also, the demand for water depends on the time of the year. The player's actions are coupled through a state equation, which describes the water levels. We solve this game using a feedback Nash equilibrium concept. However, our first solution does not satisfy (exogenous) environmental watchdog's expectations. Given an incentive scheme, we calculate another feedback Nash equilibrium that is more environmentally friendly than the previous one. The results are discussed in a general context of existence of feedback Nash equilibria in dynamic games.

About Weak Stationarity of Sets Systems
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Primal space description of the set of points satisfying general necessary extremality conditions in terms of strict Frecht subdifferentials and normals (weak stationary points) is presented. Different settings of extremal problems (in terms of sets systems and multifunctions) are considered.

The Bramble-Hilbert Lemma for Convex Domains
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The Bramble-Hilbert lemma is a fundamental result on multivariate polynomial approximation. It is frequently applied in the analysis of Finite Element Methods (FEM) used for numerical solutions of PDEs. However, this classical estimate depends on the geometry of the domain and may 'blow-up' for simple examples such as a sequence of triangles of equivalent diameter that become thinner and thinner. Thus, in FEM applications one usually requires that the mesh has 'quasi-uniform' geometry. This assumption is perhaps too restrictive when one tries to obtain estimates of nonlinear approximation methods that use piecewise polynomials.

We will show in our lecture that it is possible to obtain estimates where the constant is independent of the geometry of the domain. Thus, we will be able to apply the estimates in processes of nonlinear multivariate approximation by piecewise polynomials on families of triangulations of $\mathbb{R}^d$ into simplices without having to pay attention to how slim they may become.

Hausdorff Operators on the Hardy Space
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The behavior of Hausdorff operators on the Hardy space is studied, in both the one-dimensional case and the multivariate case. The results are given in terms of the function generating the Hausdorff operator.

Random Matrices and Geometry of Random Polytopes.

Alexander Litvak
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We establish a deviation inequality for the smallest singular value of random matrices. We provide applications to geometry of random polytopes.

Splitting Formulas for Graph and Knot Polynomials and their Algorithmic Use

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We give an overview and unify various techniques of computing graph and knot polynomials efficiently on input which satisfy various structural properties. The abstract, and only theoretically efficient, version of the technique is based on a generalization of the Feferman-Vaught theorem for Monadic Second Order Logic. Practically efficient versions include the Tutte polynomial and colored Tutte polynomials, the generating function for SAT and others.

Recent related papers may be found at http://www.cs.technion.ac.il/~admlogic/TR/readme.html

50 Years of the Spectrum Problem: Overview and New Directions

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The spectrum of a sentence (in some logic) is the set of cardinalities of its finite models. The spectrum problems asks to characterize spectra of sentences of First Order (or fragments of higher order) Logic. We review results and approaches of the last 50 years and indicate new directions.

Analyticity of Solutions to Functional Differential Equations

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We investigate solutions to functional differential equations of the form

$$y'(z) + ay(z) = by(g(z)),$$

where a,b are complex constants, and g is an entire function. Specifically, we are interested in analytic solutions about some initial point z_0 which remains fixed under g. The character of the fixed point determines whether analytic solutions arise, and the solutions then generally have a natural boundary depending only on the functional argument $g(z)$.

Holomorphic Motions

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Holomorphic motions is the theory which describes the motion of dynamically defined subsets of the plane (such as Julia sets and Limit sets of Kleinian groups) as an underlying holomorphic dynamical system is varied analyticall. Here we present a new proof, via the nonlinear Cauchy problem to the so called generalised lambda lemma of Sullivan/Thurston as first established by Slodkowski.

Online Width Metrics

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Real-life online situations often give rise to input patterns that seem to be “long” and “narrow”, that is, pathlike. One could argue that the most compelling reason for attempting to solve a problem online is that the end of the input is “too long coming” according to some criteria that we have. Given such a situation, we attempt to do the best we can with the partial information available at each timestep.

We seek to understand the effects of using graph width metrics as restrictions on the input to online problems. It seems natural to suppose that, for graphs having some form of bounded width, good online algorithms may exist for a number of natural problems. In the work presented we concentrate on online graph coloring problems, where we restrict the allowed input to instances having some form of bounded pathwidth. We also consider the effects of restricting the presentation of the input to some form of bounded width decomposition or layout.

Geometric Numerical Integration

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I will survey the young but growing field of geometric numerical integration, used in applications from mechanics, particle physics, celestial mechanics, chemistry, and quantum mechanics. My talk will concentrate not on specific algorithms or applications but on the mathematical structures and problems suggested by geometric integration.

Borel Normality, Automata, and Complexity

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The sequence selected from a sequence $R(0)R(1)\dots$ by a language L is the subsequence of all bits $R(n+1)$ such that the prefix $R(0)\dots R(n)$ is in L. By a result of Agafonoff, a sequence is normal if and only if any subsequence selected by a regular language is again normal. Kamae and Weiss, among others, have raised the question of how complex a language L must be such that selecting according to the language does not preserve normality. We show that there are such languages that are only slightly more complicated than regular ones, namely, normality is neither preserved by linear languages nor by deterministic one-counter languages. In fact, for both types of languages it is possible to select a constant sequence from a normal one. On the other hand, we investigate how easy such a normal sequence must be in terms of its computational complexity to serve as a counterexample.

Degrees of Randomness for Random Reals

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Related to the talks of Rod Downey and Liang Yu, we compare several notions of “degrees of randomness”, with specific attention to the degree of random reals. We will also explain how sufficiently random reals are similar to trivial reals (i.e., the maximally non-random reals discussed in André Nies’ talk) and how work on trivial reals has been applied to the study of degrees of randoms.

The Structure of Abelian Pro-Lie Groups

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The class of abelian pro-Lie Groups is the smallest complete category containg the class of finte-dimensional abelian Lie groups. It includes all locally compact abelian groups. The structure of abelian pro-Lie groups is described.

Demand Responsiveness in Additive Cost Sharing

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We propose two new axioms of demand responsiveness for additive cost sharing with variable demands. Group Monotonicity requires that if a group of agents increase their demands, not all of them pay less. Solidarity says that if agent i demands more, j should not pay more if k pays less. We explore their impact on the so-called full responsibility theory, postulating the standard Separability axiom, and on the partial responsibility theory postulating Strong Ranking, the requirement that the ranking of cost shares should never contradict that of demands. Under Separability, neither the Aumann-Shapley nor the Shapley-Shubik method is group monotonic; on the other hand, convex combinations of "nearby" fixed-path methods are group-monotonic: the subsidy-free serial method is the main example. No separable method meets Solidarity, yet restricting the axiom to submodular cost functions and adding the standard Monotonicity characterizes the fixed-flow methods, containing the Shapley-Shubik and serial methods. The combination of Strong Ranking (partial responsibility), Solidarity and Monotonicity characterizes the quasi-proportional methods, under which cost shares are proportional to 'rescaled' demands.

Randomness Notions and Lowness Properties

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A set of natural numbers is anti-random (or K-trivial) if the prefix complexity of all initial segments is minimal (sets are identified with infinite sequences of 0's and 1's). Given a randomness notion C, let $\mathrm{Low}(C)$ be the class of oracles which do not change C when relativizing to them. We give an introduction to recent results connecting those classes. (a)–(c) below answer questions from the literature, for instance [1].

(a) We show that, if C is Martin-Löf randomness, then $\mathrm{Low}(C)$ coincides with the class of anti-random sets [3]. Kucera (1993) studied a further class, the sets A which are computable in some Z which is random in A. It easy to see that each low for Martin-Löf random set is a Kucera set. In very recent research,

Hirschfeldt and Nies have shown that in fact each Kucera set is anti-random, improving a result in [4]. So all three notions coincide.

(b) If C is computable randomness, the only Low(C) sets are the computable ones [3].

(c) If C is Schnorr randomness, Low(C) coincides with the recursively traceable sets [2].

(d) We prove that lowness for non-monotonic (NM) random implies anti-random [3], but here no complete classification is known yet. Equivalence of NM- and Martin-Löf randomness is a major open problem.

(e) If A is anti-random, then the halting probability of any universal machine relative to A is an r.e. real number.

1. K. Ambos-Spies, A. Kucera, *Randomness in computability theory*, in: Computability Theory and Its Applications: Current Trends and Open Problems, AMS, 2000.

2. B. Kjos-Hanssen, A. Nies, F. Stephan, *On a question of Ambos-Spies and Kucera*, to appear.

3. A. Nies, *Lowness properties and randomness*, to appear.

4. A. Nies, F. Stephan, S. Terwijn, *Randomness, relativization, and Turing degrees*, to appear.

Reconstruction from Boundary Measurements

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An inaccessible domain D in three-space is filled with a inhomogeneous medium; we wish to reconstruct or to evaluate a physical characteristic of the medium from a series of measurements on the boundary B. The characteristic is a unknown function F in D (attenuation coefficient, conductivity, refraction index, density of certain elements, etc.). The information from within D is conveyed to the surface B by a physical field (photons, electrons, positrons, electric field, acoustic waves, etc.). Various numerical methods are used for reconstruction whose objective is give an algorithm which converges to a "solution". In the most important cases one can not check the reconstruction by a independent method. The accuracy of the reconstruction can not be very high and only a stable information is reliable. The problem is well-posed and the reconstruction is stable enough in three "classical" cases: plane X-ray tomography, electron microscopy and the Magnetic Resonance Imaging (A.Cormack-G.Haunsfield, Nobel Prize of 1979, A.Klug, Nobel Prize of 1982, P.Lauterbur-P.Mansfield, Nobel Prize of 2003, respectively). In the general case, we can expect that only a part of information can be reliably recovered from the boundary measurements. The mathematical problem is how to delimitate and to extract the reliable information. We discuss these questions for few ill-posed inverse problems: the electric impedance, the thermoacoustic and the diffusion tomography.

On Solvability of Functional Equations Relating to Dynamical Systems with Several Generators

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In my talk some solvability properties for functional equations of the form $F(t) - a(t)F(x(t)) - b(t)F(y(t)) = h(t)$ are considered. Here F is an unknown real-valued continuous function on a finite closed interval I, $x(t)$ and $y(t)$ are given continuous maps of I into itself, and $a(t)$, $b(t)$, and $h(t)$ are given real-valued continuous functions on I. Such equations are of interest not only by themselves as an object of analysis, but they are also a necessary link in solving various problems in such diverse fields as integral and functional equations, measure theory, integral geometry and boundary problems for partial differential equations. The major part of the proofs is based on the new results in the theory of dynamical systems generated by a noncommutative semigroup with several generators.

1. B. Paneah, *On solvability of functional equations relating to dynamical systems with two generators*, Functional Analysis and Its Applications 37 (2003), 46–60.

2. B. Paneah, *Dynamical approach to some problems in integral geometry*, Trans. Amer. Math. Soc., to appear.

3. B. Paneah, *Noncommutative dynamical systems with two generators and their applications in analysis*, Discrete and Continuous Dynamical Systems, to appear.

Modelling of Quantum Networks

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Transport properties of quantum networks are investigated using solvable models based on the introduction of an intermediate operator.

Oscillation Stability in Topological Groups

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In view of interesting recent links between geometric functional analysis and topological transformation groups (such as extreme amenability), Vitali Milman proposed to the speaker that a better understanding of the famous distortion property of the Hilbert space (Odell and Schlumprecht) may be achieved in a topological transformation group framework. The following general concept, recently suggested in a joint work of the speaker with A. Kechris and S. Todorcevic (http://arxiv.org/abs/math.LO/0305241), covers both the distortion property and the Devlin theorem in infinite combinatorics. I will present this development, which currently consists of a definition, two major examples, and two open questions.

Say that a left uniformly continuous function f on a topological group G is *oscillation stable* if for every $\epsilon > 0$ there is a right ideal J in the topological semigroup $\hat{G}^L$, the completion of G with regard to the left uniform structure, with the property that the values of the unique extension of f by continuity to any two points of J differ by $< \epsilon$. If H is a closed subgroup of G, say that the homogeneous space G/H is *oscillation stable* if for every bounded left uniformly continuous function f on G/H the composition with the factor-map is oscillation stable on G. Absence of oscillation stability is a natural analogue of distortion. Two main examples are: the sphere in the Hilbert space as a factor-space of the unitary group with the strong topology, and the set of n-subsets of rationals as a factor-space of the group of order-preserving automorphisms of $\mathbb{Q}$ with its unique Polish topology.

Two interesting open (as of 30.01.2004) questions are: (1) does there exist a topological group with at least two elements such that G is itself oscillation stable? (2) is the universal Urysohn metric space (of diameter 1) oscillation stable?

Strictly Hermitian Positive Definite Functions

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We talk about characterizations of various classes of positive definite and Hermitian positive definite functions. In particular we are interested in when $f(< x, y >)$ is a (Hermitian) positive definite, and strictly (Hermitian) positive definite function for x, y in H, where H is an arbitrary (complex) inner product space.

A New Algorithm for Unconstrained Minimization without Derivatives

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The curvature of the objective function is important in unconstrained minimization calculations. Therefore many algorithms form quadratic approximations to the objective function, and then a new vector of variables is generated by minimizing the current quadratic model subject to a bound on the length of the step from the best vector so far. Each approximation may be defined by interpolation to (n+1)(n+2)/2 values of the objective function, where n is the number of variables, or fewer conditions can be combined with a suitable way of taking up the freedom in each new quadratic model. The new algorithm minimizes the Frobenius norm of the change to the second derivative matrix of the model. It provides excellent numerical results when the number of interpolation conditions is only 2n+1, including some cases with 200 variables.

Concentration of Measure in Banach Spaces and Criteria for Amenability

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We will look at some characterisations and criteria for amenability and how they are related to concentration in high dimensional objects. Some examples and counter-examples shall also be presented.

Multi-Linear Formulas for Permanent and Determinant are of Super-polynomial Size

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Arithmetic formulas for computing the permanent and the determinant of a matrix have been studied since the 19th century. Are there polynomial size formulas for these functions? Although the permanent and the determinant are among the most extensively studied computational problems, polynomial size formulas for these functions are not known. An outstanding open problem in complexity theory is to prove that polynomial size formulas for these functions do not exist. We solve this problem for the subclass of multi-linear formulas:

An arithmetic formula is multi-linear if the polynomial computed by each of its sub-formulas is multi-linear, that is, in each of its monomials the power of every input variable is at most one. Multi-linear formulas are restricted, as they do not allow the intermediate use of higher powers of variables in order to finally compute a certain multi-linear function. Note, however, that for

many multi-linear functions, formulas that are not multi-linear are very counter-intuitive. Note also that both the permanent and the determinant are multi-linear functions and that many of the well known formulas for these functions are multi-linear formulas.

We prove that any multi-linear arithmetic formula for the permanent or the determinant of an n dimensional matrix is of size super-polynomial in n.

Locally Convex Cones as Generalisations of Locally Convex Vector Spaces

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Endowed with suitable topologies, vector spaces yield rich and well-studied structures. Locally convex topological vector spaces in particular permit an expensive duality theory whose study gives valuable insight into the spaces themselves. Some important mathematical settings, however, while close to the structure of vector spaces do not allow subtraction of their elements or multiplication by negative scalars. Examples are certain classes of functions that may take infinite values or are characterized through inequalities rather than equalities. They arise naturally in integration and in potential theory. Likewise, families of convex subsets of vector spaces which are of interest in various contexts, do not form vector spaces. If the cancellation law fails, domains of this type may not even be embedded into larger vector spaces in order to apply results and techniques from classical functional analysis. They merit the investigation of a more general structure. The theory of locally convex cones includes most of these settings. A topological structure on a cone is introduced using order theoretical concepts. Staying reasonably close to the theory of locally convex spaces, this approach yields a sufficiently rich duality theory including Hahn-Banach type extension and separation theorems for linear functionals. We shall present an outline of the principal concepts of this emerging theory, survey some the main results and provide primary examples and applications.

Monotonic Analysis

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Examination of many problems arising in optimisation and related topics can be successfully accomplished if these problems enjoy a convex structure. Convexity is, however, a very restrictive hypothesis so the question arises: is it possible to substitute convexity for another structure at least for some broad classes of non-convex problems? Monotonicity is one of such structures. The main notions of convex analysis (subdifferentials, Fenchel-Moreau conjugacy, inf-convolution etc.) and results related to them play a key role in applications of convexity. Thus the question arises: "Is it possible to develop a theory of monotonicity similar in some aspects to convex analysis?" We use the term monotonic analysis for this emerging theory. Monotonic analysis can be considered as abstract convex analysis based on special classes of elementary functions. Convex problems are usually much simpler than problems with monotonic data however some theoretical schemes of convex duality and some numerical schemes of convex programming can be successfully applied also in the monotonic setting. In this lecture we present some recent developments in the area of monotonic analysis.

The Hahn-Banach Theorem and Maximal Monotonicity
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We introduce a generalized form of the Hahn-Banach theorem, which we will use to prove various results on the existence of linear functionals in functional analysis, convex analysis and optimization, and also to prove a minimax theorem. We apply our results to the "Fitzpatrick function" to obtain criteria for a monotone multifunction, T, on a reflexive Banach space to be maximal monotone, with a sharp lower bound on the solutions, x of the equation "0 in $(T + J)x$". We do not use any renorming theorems, any fixed-point theorems, or any result that depends on Baire's theorem.

An Overview on Recent Degree-Theoretic Results in Kolmogorov Complexity and Algorithmic Randomness
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Coauthors: Parts of the talk are based on joint papers with Richard Beigel, Harry Buhrman, Peter Fejer, Lance Fortnow, Piotr Grabowski, Luc Longpre, Andrej Muchnik, Andre Nies, Sebastiaan Terwijn and Leen Torenvliet.

The author gives an overview on his recent work in the fields of Kolmogorov complexity and Algorithmic randomness. He presents the following results.

A set is called Martin-Löf random iff there is a constant c with $H(x) > |x| - c$ for all prefixes x of the set where H is the prefix-free Kolmogorov complexity. The author shows that the Martin-Löf random sets are either Turing hard for the halting problem K or do not have a Peano-complete degree. Thus their degrees are not closed upward.

Schnorr-random sets are the most common generalization of Martin-Löf random sets. It is shown that the Turing degrees of Schnorr-random sets are those of Martin-Löf random sets plus the high degrees. Every high degree contains a set which is Schnorr-random but not Martin-Löf random while in every non-high degree these two notions coincide.

A set is called Kolmogorov-random if $C(x) > |x| - c$ for a constant c and infinitely many prefixes x of the set. The Kolmogorov random sets coincide with the sets which are Martin-Löf random relative to the halting problem oracle K.

A function f is k-enumerable iff there is an algorithm which enumrates for every x up to k values such that one of them is $f(x)$. Several natural functions like the k-fold characteristic function and the k-fold cardinality function of a set are known not to be k-enumerable. These results are extended to the Kolmogorov function which assigns to any x the Kolmogorov complexity $C(x)$ of x: If the Kolmogorov function is k-enumerable relative to an oracle A for a constant k then A is already Turing hard for the halting problem K. In particular it is as hard to k-enumerate the Kolmogorov function as to compute it directly with the given oracle.

Quasi-metric Spaces with Measure

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The phenomenon of concentration of measure on high dimensional structures is usually stated in terms of a metric space with a Borel measure, also called an mm-space. We extend some of the mm-space concepts to the setting of a quasi-metric space with probability measure (pq-space). Our motivation comes from biological sequence comparison: we show that many common similarity measures on biological sequences can be converted to quasi-metrics. We show that a high dimensional pq-space is very close to being an mm-space.

Graph Algebras and Noncommutative Spaces

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We outline some recent constructions of quantum deformations of certain compact manifolds which are related to the generalized Cuntz-Krieger algebras of directed graphs.

Geometric Inequalities for a Class of Exponential Measures

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Using so-called M-ellipsoids introduced by V. Milman in the mid-1980's we prove versions of the inverse Santaló inequality and the inverse Brunn-Minkowski inequality for a general class of exponential measures. Originally, these inequalities were proved for the usual volume on R^n by Bourgain and Milman, and by Milman, respectively. The class of measures considered here contains in particular the Gaussian measure on R^n.

Applications of FDE to Cell/Tumour Growth

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Cell population cohorts in which the cells are simultaneously growing and dividing, are known to satisfy a non-local linear parabolic partial differential equation (Wake, et al, Comm. Appl. Analysis 2000). The space-like variable is the size-structure or, equally useful, DNA content. These equations admit seperable solutions, which are globally attracting as time increases. This provides a simple and powerful model for the experimentall observed fact: that cell population cohorts evolve according to a "steady-size-distribution" (SSD, SDD attribute). The time constants of such an evolution are eigenvalues of a non-local singular Sturm-Liouville problem.

This attribute enables remarkably simple and useful models to be constructed which enable growth characteristics to be determined for both healthy and diseased scenarios. This framework has been successfully applied by the author and his co-workers (B. Basse, B. van-Brunt, D. Wall and B. Baguley) to underpin decision support for cells in plant roots, plankton, tumours and muscle growth. The SSD attribute was shown recently (Basse et al, J.Math.Biol, 2003) to hold for multi-compartment scenarios — wherein the cells were structured into compartments reflecting the four or more steps of the cell cycling process. In particular this enables the effects of intervention such as cancer therapy by drugs or radiation to be quantified.

A Spectral Dichotomy for the Koopman Operator

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We will discuss the 1 parameter family of unitary operators associated to a measure preserving flow. We consider the case where the flow is a time-change of an irrational linear flow. In this case the eigenvalues of the family depend on a subtle interaction between the time-change and the arithmetic properties of the translation. We find a class of timechanges for which the operator has either pure-point spectrum or continuous spectrum depending upon the irrational translation.

A Structural Equivalence for the Determinacy of Real Games

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In a variety of restricted settings, structural equivalences for the determinacy of integer games have been identified. The restriction is in terms of limiting the domain of consideration to fairly specific cases such as $L(R)$. Without these restrictions the problem is much more difficult. We discuss recent results on the outright equivalence for the determinacy of real games. While this is a stronger determinacy hypothesis there is no further restriction assumed.

Degrees of D.C.E. Reals

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A real is called c.e. if it is the halting probability of a prefix free Turing machine. Equivalently, a real is c.e. if it is left computable in the sense that $L(\alpha) = \{q \in \mathbb{Q} : q \leq \alpha\}$ is a computably enumerable set. The natural field formed by the c.e. reals turns out to be the field formed by the collection of reals of the form $\alpha - -\beta$ where α and β are c.e. reals. While c.e. reals can only be found in the c.e. degrees, Zheng has proven that there are Δ^0_2 degrees that are not even n-c.e. for any n and yet contain d.c.e. reals. We prove that every ω-c.e. degree contains a d.c.e. real, but there are $\omega + 1$-c.e. degrees, and hence Δ^0_2 degrees, containing no d.c.e. real.

Hybrid Steepest Descent Method for the Variational Inequality Problem over the Fixed Point Sets of Certain Quasi-nonexpansive Mappings

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The hybrid steepest descent method is an algorithmic solution to the variational inequality problem over the fixed point set of nonexpansive mapping and applicable to broad range of convexly constrained nonlinear inverse problems in real Hilbert space.

In this talk, we show that the strong convergence theorem of the method for nonexpansive mapping can be extended naturally to a strong convergence theorem of the method for the variational inequality problem over the fixed point set of attracting quasi-nonexpansive mapping. By this generalization, we can approximate successively to the solution to the convex optimization problem over the fixed point set of subgradient projection operator in real Hilbert space.

Reducible Convex Sets

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A compact convex set P, symmetric about the origin, is said to be reducible if there is another compact convex set Q, not symmetric about any point, such that $P = \{x - y : x, y \in Q\}$. For example, a regular hexagon is reducible because it is the difference set of a triangle. In fact, every 2-dimensional such P, other than a parallelogram, is reducible. This is not so in higher dimensions, where "most" convex bodies turn out to be irreducible. Connections with decomposability of convex sets, best approximation and quasilinear functions may also be indicated.

Randomness

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I will give a talk about some recently exciting results of randomness.

The Reduced Birman-Murakami-Wenzl Algebra of Type B_n

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There has been a lot of interest in the representation theory of Birman-Murakami-Wenzl (BMW) algebras, which are related to Artin braid groups (of type A_n). Recently, analogues of these algebras have been investigated for other Coxeter types. The focus of this talk will be the reduced BMW algebra of type B_n, as defined by Häring-Oldenburg. We will present a result regarding the dimension of this algebra.

Covering Banach Spaces

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The following generalization of Corson's theorem will be stated. Let a Banach space X admit a locally finite covering by w-closed bounded sets: then X is saturated by c_0. As a consequence, we answer in the negative to a question posed by V. Klee in 1981. In fact, for any infinite cardinal number n, space $l^1(n)$ does not admit a locally finite covering by bounded w-closed sets (e.g. by balls which are in the Klee's question mentioned above).